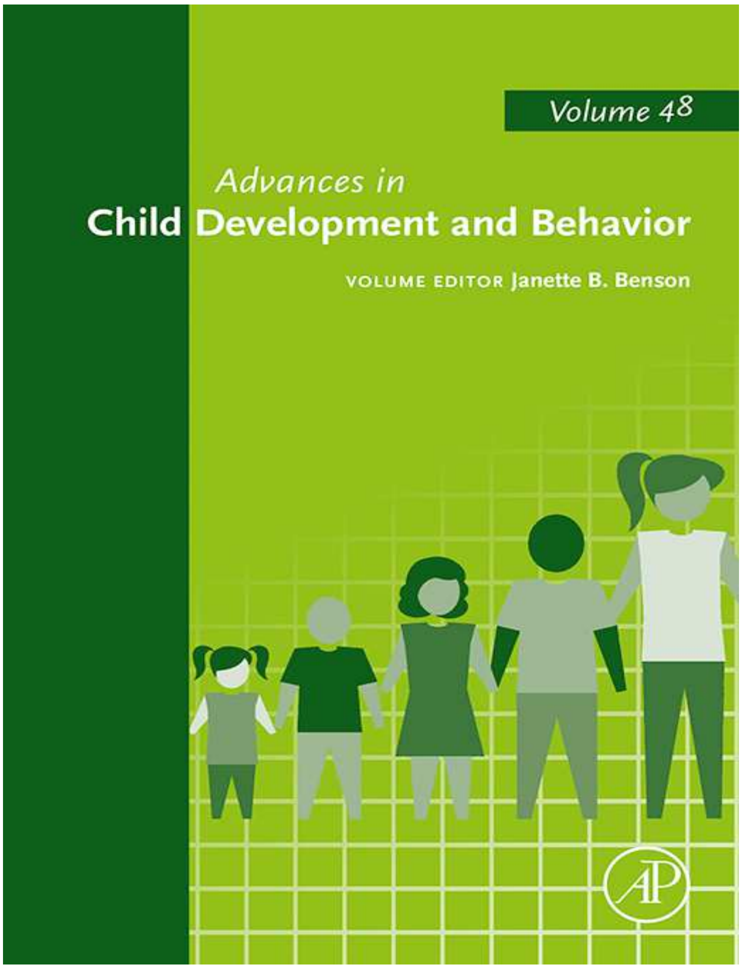

From Ian M. Lyons and Daniel Ansari, Foundations of Children's Numerical and Mathematical Skills: The Roles of Symbolic and Nonsymbolic Representations of Numerical Magnitude. In: Janette B. Benson, editor, *Advances in Child Development and Behavior, Vol. 48*, Burlington: Academic Press, 2015, pp. 93-116.
ISBN: 978-0-12-802178-1

Academic Press

CHAPTER THREE

Foundations of Children's Numerical and Mathematical Skills: The Roles of Symbolic and Nonsymbolic Representations of Numerical Magnitude

Ian M. Lyons, Daniel Ansari[1]
Numerical Cognition Laboratory, Department of Psychology & Brain and Mind Institute, University of Western Ontario, London, Ontario, Canada
[1]Corresponding author: e-mail address: daniel.ansari@uwo.ca

Contents

Abstract

Numerical and mathematical skills are critical predictors of academic success. The last three decades have seen a substantial growth in our understanding of how the human mind and brain represent and process numbers. In particular, research has shown that we share with animals the ability to represent numerical magnitude (the total number of items in a set) and that preverbal infants can process numerical magnitude. Further research has shown that similar processing signatures characterize numerical magnitude processing across species and developmental time. These findings suggest that an approximate system for nonsymbolic (e.g., dot arrays) numerical magnitude representation serves as the basis for the acquisition of cultural, symbolic (e.g., Arabic numerals) representations of numerical magnitude. This chapter explores this hypothesis by reviewing studies that have examined the relation between individual differences in nonsymbolic numerical magnitude processing and symbolic math abilities (e.g., arithmetic). Furthermore, we examine the extent to which the available literature provides strong evidence for a link between symbolic and nonsymbolic representations of numerical magnitude at the behavioral and neural levels of analysis. We conclude that claims that symbolic number abilities are grounded in the approximate system

Advances in Child Development and Behavior, Volume 48
ISSN 0065-2407
http://dx.doi.org/10.1016/bs.acdb.2014.11.003

for the nonsymbolic representation of numerical magnitude are not strongly supported by the available evidence. Alternative models and future research directions are discussed.

1. INTRODUCTION

Numerical information informs our everyday behavior. Consider a glance at the alarm clock in the morning, counting change in the line-up at the coffee shop, or reading about the latest election polls—each of these common situations places demands on the ability to process numerical information. Research has shown that the ability to process numbers and use them in mathematical operations (such as calculation) is a critical predictor of an individual's economic and social success (e.g., Bynner & Parsons, 1997). Longitudinal studies investigating the predictors of academic achievement reveal that school-entry numerical and mathematical skills are a strong predictor of later academic achievement. School-entry math skills are a stronger predictor of later achievement than both school-entry reading and attentional skills (e.g., Duncan et al., 2007). Findings such as these demonstrate the critical role that numerical and mathematical knowledge and skills play in children's academic development and outcomes.

What do we know about how numbers are represented in the brain and mind and how such representations change over the course of learning and development? The past three decades have seen a surge in the empirical study of number representation and processing in multiple species and at different levels of analyses (for reviews, see Ansari, 2008; Dehaene, 1997; Nieder & Dehaene, 2009). The aim of this chapter is to provide an overview of this research and to synthesize what is currently known, as well as to discuss open questions and future research directions.

2. AN APPROXIMATE SYSTEM FOR THE REPRESENTATION OF NUMERICAL MAGNITUDE

Much of the research on how we represent and process numerical information has been focused on uncovering the foundational systems that underpin the development of complex numerical and mathematical abilities. In particular, there has been a focus on understanding the representations of numerical magnitude, or the total number of items in a set. The representation of processing numerical magnitude has been investigated from infancy onward at both the behavioral and brain levels of analysis. In addition to studies with humans, numerical magnitude processing has been investigated in nonhuman primates in an effort to investigate cross-species similarities and differences in processing (Dehaene, 1997; Nieder & Dehaene, 2009). These studies have revealed that there are similarities in the way in which non-human primates, human infants, children, and adults represent and process numerical magnitude. This convergence across species and developmental time has lead to the suggestion that humans are born with an approximate system for the processing of numerical magnitude.

Data from looking time paradigms have demonstrated that young infants can discriminate between numerical quantities. In a seminal study, Xu and Spelke (2000) found that 6-months-old infants discriminate between an array of 8 dots and an array of 16 dots, but not between 8 versus 12 dots. These results not only demonstrated for the first time that young infants can discriminate between two arrays of relatively large numerical magnitudes but also that their ability to do so is influenced by the numerical ratio (smaller/larger) between the numerical quantities. This pattern of results has been widely replicated, and the precision with which infants can discriminate between numerical magnitudes improves over the first year of life (see Libertus & Brannon, 2009 for a review of the infant numerical magnitude processing literature).

The influence of numerical distance and ratio on numerical magnitude discrimination has been demonstrated in numerous studies with both human children and adults (e.g., Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977). Specifically, when children and adults judge which of two numerical magnitudes (either numerical symbols, such as Arabic numerals or non-symbolic stimuli, such as dot arrays, see Figure 1) are numerically larger, the speed and accuracy of their judgments is correlated with the numerical distance, and ratio of the numerical magnitude they compare. For children and adults, the larger the numerical distance between the magnitudes (or the closer the ratio between them is to 1), the slower and more error-prone their judgments of relative numerical magnitude are (see Figure 2).

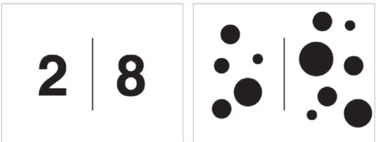

Figure 1 Symbolic and nonsymbolic versions of the numerical magnitude comparison task.

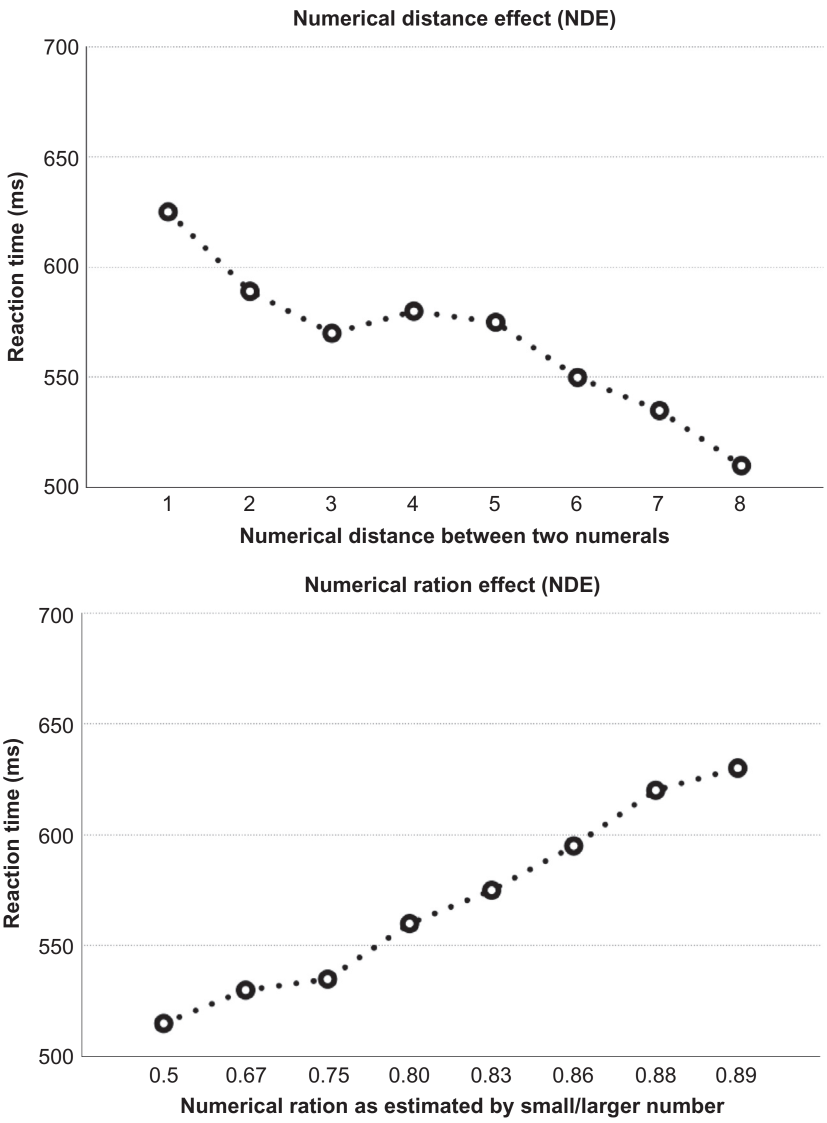

Figure 2 Examples of the numerical distance effect (NDE) (A) and the numerical ratio effect (NRE) (B).

As noted by Moyer and Landauer (1967) in their seminal paper, the numerical ratio explains more variability in numerical magnitude comparison data than does the numerical distance. Take, for example, the comparisons of a stimulus display of 8 versus 9 and a display of 1 versus 2. Both of these number pairs have a numerical distance of 1 but their ratio is different.

Thus, a numerical distance effect would predict similar response times for both pairs, while the ratio effect predicts longer reaction times for 8 versus 9 compared to 1 versus 2 (i.e., 1 is 50% of 2, while 8 is 88% of 9). Despite the better prediction of reaction time data during comparison when ratio rather than distance is the independent variable, distance and ratio are highly correlated; thus, very similar predictions result in slight differences in the variance explained, rather than representing radically different models of numerical magnitude comparison data.

In addition to the demonstrated existence of qualitatively similar distance and ratio effects in studies with humans ranging from young infants to adults, the influence of distance/ratio on numerical magnitude discrimination has also been shown in studies of nonhuman primates (e.g., Brannon & Terrace, 1998; Cantlon & Brannon, 2006).

This body of evidence has lead to the proposal that humans share with other species an approximate system for the representation and processing of numerical magnitude, and given findings from infants, that humans are born with such a system (for a review, see Cantlon, 2012). It is because of the effect of numerical distance/ratio that the system is purported to represent numerical magnitude approximately. That is, if numerical magnitudes were represented exactly, then the ability to discriminate between them should not vary as a function of their similarity (i.e., as a function of distance/ratio), which is of course not the case. The fact that similarity between numbers predicts how well they can be differentiated from one another suggests that numbers that are close are represented more similarly than those that are comparatively further apart. Moreover, the existence of the numerical ratio effect suggests that the similarity between numerical magnitudes will increase as a function of their size. For example, the similarity between 9 and 8 is greater than that between 1 and 2. In this way, nonsymbolic numerical magnitudes are thought to be represented in an analog rather than digital format (Lyons, Ansari, & Beilock, in press). The most prominent account of the numerical distance/ratio effects posits that numbers are represented along a mental continuum (a "mental number line") and that the representations of numerical magnitude overlap with one another, but that their overlap decreases as a function of the numerical distance/ratio between them (see Figure 2). While some have argued that numbers are represented on a linear scale (Figure 3, panel A), other researchers contend that the Gaussian tuning curves have a fixed width, but are represented along a logarithmic number line (Figure 3, panel B). It is beyond the scope of this chapter to discuss these subtle differences in the way in which the analog representation

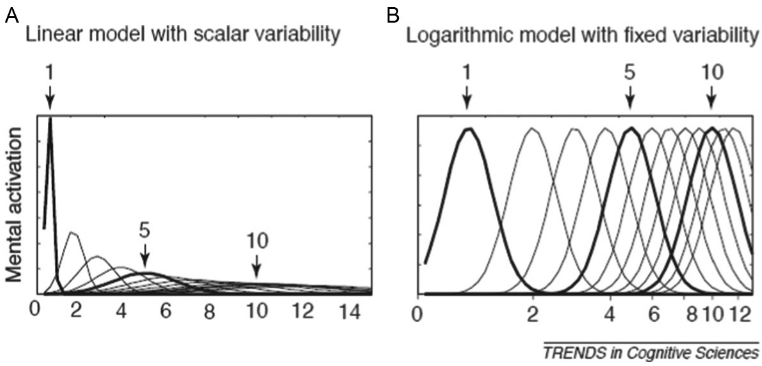

Figure 3 Models of the approximate representations of numerical magnitude.

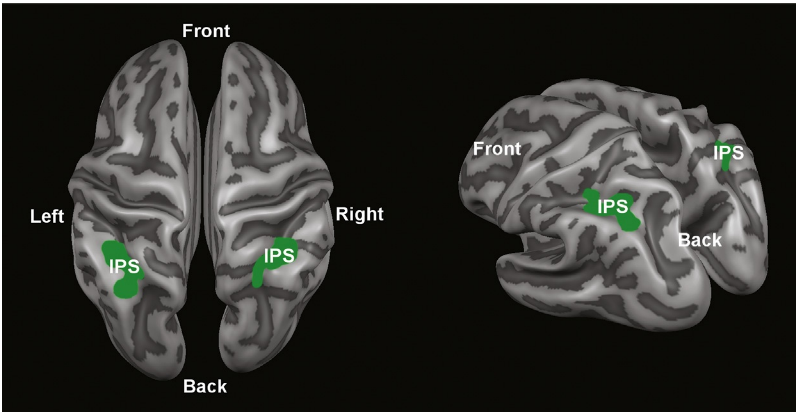

Figure 4 Reconstruction of the human brain, showing the intraparietal sulcus (IPS) displayed in green (light gray in the print version).

of numerical magnitude has been conceptualized (for a review, see Feigenson, Dehaene, & Spelke, 2004). It is important to note in the context of the current discussion that representations are thought to be approximate/analog in both models. Thus according to this account, the observation of ratio/distance effects across species and human developmental time indicates the existence of an approximate system of numerical magnitude representation that is likely evolutionarily ancient (given the evidence from nonhuman primates) and innate (given the evidence from very young human infants).

In addition to behavioral evidence (such as looking times or reaction times), recent advances in noninvasive neuroimaging have allowed researchers to search for the neural correlates of numerical magnitude representations. Researchers have found that a brain region referred to as the parietal cortex, and in particular the intraparietal sulcus (see Figure 4), plays a key role in numerical magnitude processing (for reviews, see Ansari, 2008;

Nieder & Dehaene, 2009). Moreover, studies with infants and young children have revealed that this brain region is activated during numerical magnitude processing from an early age onward (Cantlon, Brannon, Carter, & Pelphrey, 2006; Hyde, Boas, Blair, & Carey, 2010; Izard, Dehaene-Lambertz, & Dehaene, 2008). Thus, brain imaging provides another source of evidence in support of the notion that an approximate system for the representation of numerical magnitude exists from an early age onward and that there are qualitative similarities in the representation of numerical magnitude over the course of development.

3. THE SYMBOLIC REPRESENTATION OF NUMERICAL MAGNITUDE

The literature just reviewed suggests the existence of an approximate representation of numerical magnitude that humans share with other species and that can be measured very early in human development. All of the findings reviewed thus far relied upon nonsymbolic representations of numerical magnitude (e.g., dot arrays) to glean insights into the representation and processing of numerical magnitude. In contrast to nonhuman primates, however, humans who grow up in literate cultures acquire symbolic representations of numerical magnitude, such as number words and Arabic numerals. Arabic numerals are something of a quasi-universal language of mathematics, since they are used to represent numerical magnitude across the globe. This raises a key question: Is the human acquisition of symbolic representations of numerical magnitude grounded in the approximate, potentially innate nonsymbolic representations of numerical magnitude?

Various researchers have hypothesized that children's approximate, nonsymbolic numerical magnitude processing abilities form the foundation on which more sophisticated, symbolic, culturally acquired mathematical skills rest (e.g., Dehaene, 1997).

If this hypothesis is correct, one would expect that a child with a representation of nonsymbolic numerical magnitude (e.g., one that is especially accurate) would be more likely to excel on mathematical tasks, such as standardized tests of symbolic arithmetic and other math abilities. In fact, several recent papers have shown just that: Individual differences in adults', children's, and infants' ability to process nonsymbolic numerical magnitudes relate to performance on a wide range of formal math achievement tests (Bonny & Lourenco, 2013; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Gilmore, McCarthy, & Spelke, 2010; Gray & Reeve, 2014; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011, 2013; Libertus, Odic, & Halberda, 2012; Lonnemanna, Linkersdörfera, Hasselhorna, & Lindberg, 2011; Lourenco, Bonny, Fernandez, & Rao, 2012; Lyons & Beilock, 2011; Mazzocco, Feigenson, & Halberda, 2011a,b; Piazza et al., 2010; Starr, Libertus, & Brannon, 2013; vanMarle, Chu, Li, & Geary, 2014; for a review, see Feigenson, Libertus, & Halberda, 2013). To measure nonsymbolic numerical magnitude processing ability, many of these studies ask participants to determine which of two arrays of objects (typically dots on a computer screen) contains more objects (right panel of Figure 1). Arrays are typically presented too quickly for participants to count the dots individually, so they must instead rely on their intuitive sense of approximate magnitude. How well a person is able to complete this task is taken as a measure of the strength of their number sense. Using this method, for example, Halberda et al. (2012) showed that individuals with stronger (or more precise) representations of nonsymbolic numerical magnitude tend to report better math achievement scores—an effect that remains stable more or less throughout the life span and exists even after controlling for achievement scores in nonmathematical domains. In a similar vein, Libertus et al. (2011) showed that preschoolers' number sense predicted math scores at the onset of formal math instruction and that performance on a dot-comparison task predicted college students' math scores on a college entrance exam (i.e., their math SAT scores; Libertus et al., 2012).

It is important to note, however, that these correlations do not disclose the causal direction between number sense and mathematical achievement. Seeking thus to draw a stronger causal claim, researchers have recently shown that approximate arithmetic training (e.g., estimating the sum of two or more dot arrays) improves symbolic math achievement scores in both adults (Park & Brannon, 2013, 2014) and in children (Hyde, Khanum, & Spelke, 2014). Taken together, these studies provide indirect evidence to suggest that a child's (possibly innate) ability to represent nonsymbolic magnitudes forms an initial footing from which more sophisticated math abilities develop. In other words, these results lend some confidence to the exciting possibility that evolutionarily ancient neural systems (the approximate number system, in this case) are co-opted to help shape the way that cultural inputs (such as number symbols) underpin more sophisticated cognitive abilities (such as mathematics) (Dehaene & Cohen, 2007).

On the other hand, considerable caution may still be warranted. First, it remains unclear precisely how this process occurs. Other studies have shown that training regimes using dot-comparison tasks fail to improve symbolic math performance (Dewind & Brannon, 2012; Park & Brannon, 2014; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). This raises the distinct possibility that it is improved *manipulation* of nonsymbolic magnitudes (in an explicitly arithmetic context), and not simply increased precision of said magnitudes, that leads to improvement in symbolic math ability (see also Park & Brannon, 2014). Such a distinction may have crucial implications for early education: it may not be enough to simply expose young children to nonsymbolic magnitudes; rather, the benefits of such exposure may depend crucially on activities that expressly encourage children to manipulate those magnitudes in a mathematical context.

Furthermore, a more detailed look at the correlations between nonsymbolic magnitude processing and math achievement shows that this effect has proven to be inconsistently replicated (De Smedt, Noël, Gilmore, & Ansari, 2013). De Smedt et al. (2013) reviewed 25 different studies (18 with children and 7 with adult participants) and found that only a minority (7 of 18 with children, 4 of 7 with adults) showed a statistically significant relation between dot-comparison and symbolic math performance.[1] This is quite likely due to the fact that the overall size of this effect is relatively small, as revealed by a recent meta-analysis (with an average r of about 0.20 for cross-sectional studies and 0.17 for longitudinal studies; Chen & Li, 2014).

Perhaps most remarkable is the array of studies that have now shown that this relation is substantially reduced, or even entirely eliminated, once one controls for basic *symbolic* number processing abilities (Bartelet, Vaessen, Blomert, & Ansari, 2014; Brankaer, Ghesquière, & De Smedt, 2014; Castronovo & Göbel, 2012; Fazio, Bailey, Thompson, & Siegler, 2014; Fuhs & McNeil, 2013; Göbel, Watson, Lervåg, & Hulme, 2014; Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Lyons & Beilock, 2011; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014; Toll & Van Luit, 2014; vanMarle et al., 2014).

However, due to the highly variegated nature of these studies, it may be difficult to draw clear conclusions about the intervening role of basic symbolic number processing abilities. Different studies have employed different

[1] By contrast, the authors reviewed 17 studies that measured the relation between symbolic number comparison and symbolic math—13 of which showed a significant effect.

types of symbolic number tasks (e.g., number comparison, counting, or number ordering), which makes it unclear just which symbolic number skills are most crucial for underpinning more sophisticated math skills, such as mental arithmetic. Furthermore, different studies focus on different age ranges, which may add confusion because different skills may be more relevant at different points in development. Finally, not all studies control for the same factors—some might control for reading ability, basic processing speed, executive functioning, some combination thereof, or even none of these factors.

To address this issue, Lyons et al. (2014) reported data from a single, large sample spanning six academic grades (1–6, with over 200 children in each grade), that included eight different numerical tasks, standardized mental arithmetic ability, as well as three control tasks—all of which were administered to all children in the sample. The authors were thus able to examine how a wide range of basic numerical abilities relate to mental arithmetic at several different time-points in development, all while controlling for nonnumerical factors (reading ability, basic processing speed, executive functioning, as well as within-grade age variation). Highly consistent with the Chen and Li (2014) meta-analysis, Lyons et al. (2014) showed an average zero-order correlation between nonsymbolic magnitude processing (i.e., dot comparison) and mental arithmetic ability of about $r=0.24$, which was statistically significant in each grade. However, after controlling for the other seven basic numerical tasks, including several symbolic tasks, these correlations were all rendered nonsignificant. By contrast, the symbolic processing tasks remained significant predictors of mental arithmetic ability, indicating that these symbolic abilities are more directly linked to more complex math processing than is nonsymbolic magnitude processing. Interestingly, the type of symbolic processing that best predicted arithmetic ability systematically changed with age: comparing relative symbolic quantities was more predictive in younger children (grades 1–2), whereas assessing relative order of symbolic quantities was more predictive in older children (starting in grade 3 and increasing thereafter through grade 6). Note also that these results remained significant even after controlling for the fact that the symbolic and arithmetic tasks are presented in the same visual format (i.e., Indo-Arabic numerals; see also Lyons & Beilock, 2011, for a similar result in adults). This suggests that it is not just symbolic number representation *per se* that is crucial for arithmetic, but *how* these symbols are used—and the critical symbolic skills may well shift over the course of development. In sum, a more direct and fruitful approach to understanding the emergence of sophisticated arithmetic abilities may be better focused on how children learn to understand and manipulate number symbols.

A critical outstanding question is how young children first come to understand the numerical meaning of number symbols. Here, it is again tempting to assume that symbolic number understanding is bootstrapped from nonsymbolic magnitudes. However, even in kindergarteners and preschoolers, the current literature remains mixed. For instance, one may contrast a list of studies confirming a relation between nonsymbolic and symbolic number processing in kindergarten or younger children (Bonny & Lourenco, 2013; Gilmore et al., 2010; Gray & Reeve, 2014; Libertus et al., 2013; Mazzocco et al., 2011b; Starr et al., 2013) with a similar list of studies showing either the opposite (Bartelet et al., 2014; Fuhs & McNeil, 2013; Kolkman et al., 2013; Sasanguie, Defever, Maertens, & Reynvoet, 2014; Toll & Van Luit, 2014) or that basic symbolic number processing may provide the crucial intermediary step (vanMarle et al., 2014). Moreover, Mussolin, Nys, Content, and Leybaert (2014) have even provided evidence to suggest that developmental influence runs in the opposite direction—that improvement in symbolic number abilities predicts later accuracy in nonsymbolic magnitude comparison (see also Gelman & Gallistel, 2004). Specifically, Mussolin et al. (2014) measured 3–4-year-old children's ability to process symbolic and nonsymbolic numbers at two different time-points 7 months apart (41 children successfully completed all tasks at both time-points). Results showed that symbolic performance at the first time-point predicted nonsymbolic performance at the second time-point, but not the other way around. In sum, though it is certainly tempting to conclude that the cultural acquisition of sophisticated number–symbol systems operates by co-opting a more evolutionarily ancient system of nonsymbolic magnitude representation, it remains poorly understood both whether and precisely how this process may occur. It may be that only a large scale study similar to Lyons et al. (2014), but longitudinal in design and beginning with children whom have yet to receive any formal schooling, can adequately lay the issue to rest.

4. THE RELATIONSHIP BETWEEN SYMBOLIC AND NONSYMBOLIC REPRESENTATIONS OF NUMERICAL MAGNITUDE

The mixed evidence concerning the association between nonsymbolic numerical magnitude processing and children's arithmetic achievement casts doubt on the assumption that symbolic number skills are scaffolded on their nonsymbolic counterparts. Specifically, one influential proposal suggests that the very meaning of number symbols is determined by direct reference to the corresponding nonsymbolic magnitude: "When we learn number symbols, we simply attach their arbitrary shapes to the relevant nonsymbolic quantity representations" (Dehaene, 2008, p. 552; see also, Dehaene, 1997; Gallistel & Gelman, 2000). As noted earlier, the appeal of this proposal is straightforward: the solution to the mystery of how number symbols ground their meaning in reality is that they are simply linked to their evolutionary precursors—i.e., the (possibly innate) representation of the corresponding nonsymbolic magnitude. Support for this view has been consistently echoed (e.g., Eger et al., 2009; Feigenson et al., 2004, 2013; Hubbard et al., 2008; Libertus & Brannon, 2009; Lyons & Ansari, 2009; Nieder & Dehaene, 2009; Piazza, Pinel, Le Bihan, & Dehaene, 2007), and the idea has been given explicit form in the computational model proposed by Verguts and Fias (2004). To understand their model, one can imagine different internal nodes, each of which represents a given number. If a given node shows the greatest degree of activity, then the model will "respond" with the number thus indicated. Consistent with the notion of a Gaussian tuning curve along a mental number line (discussed earlier; see also Figure 3), a nonsymbolic magnitude (such as an array of dots) will tend to maximally activate the node corresponding to the correct number of dots (e.g., "six"). Note that the surrounding nodes ("five" and "seven") will also be activated—but crucially, on average, activation at these nodes will be less than at "six." Continuing the pattern, "four" and "eight" will be activated, but even less so; and so on. The idea is that the underlying representation for a given nonsymbolic magnitude is not an exact quantity, but a probabilistic representation centered on the actual magnitude (this representation is thought to drive the numerical distance and ratio effects—see Section 1). Adding random perturbations to the model causes it to generate errors that mirror human behavior: The model will most often respond "six" to six dots, but it will sometimes respond "five" or "seven" and even occasionally "four" or "eight". For a symbolic input (e.g., "6"), the model simply draws a direct link to the "six" node. In this way, symbolic representation is much more precise—both the model and humans make very few errors when dealing with symbolic inputs. The crucial point, however, is that the central node for six dots "six" is exactly the same node that "6" points to. It is in this way that the Verguts and Fias model makes explicit the view that symbolic and nonsymbolic numerical stimuli point to essentially the same underlying representation.

Three lines of evidence are typically cited in support of the proposal that symbolic and nonsymbolic numerical stimuli draw from the same underlying representations. First, as discussed in the previous section, individual differences in nonsymbolic magnitude processing are related to more complex symbolic math abilities. Second, behavioral and neural signatures such as the distance and ratio effects (see Figure 2) are qualitatively similar for symbolic and nonsymbolic numbers (e.g., Buckley & Gillman, 1974; Dehaene, 2008). Third, neural evidence has pointed to similar substrates in the brain for symbolic and nonsymbolic number processing (Diester & Nieder, 2007, 2010; Eger et al., 2009; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Piazza et al., 2007; though see Shuman & Kanwisher, 2004, for evidence to the contrary).

Recently, several counterarguments to this proposal—and to the three lines of evidence outlined in the preceding paragraph—have emerged. First, with respect to the relation between individual differences in nonsymbolic processing and arithmetic, we reviewed several lines of evidence in the previous section that casts substantial doubt on the reliability, specificity, and meaning of this relation.

Second, it is worth pointing out that ratio and distance effects are hallmarks of essentially any discrimination task, and is true for stimuli ranging from letter comparisons (which letter comes later in the alphabet; Van Opstal, Gevers, De Moor, & Verguts, 2008), to discriminations along the most basic of perceptual dimensions (such as odor discrimination in *Drosophila*; Parnas, Lin, Huetteroth, & Miesenböck, 2013), to discriminations along relatively abstract, categorical variables (such as distinguishing between species of animal figures; Gilbert, Regier, Kay, & Ivry, 2008). Of course, one would be hard pressed to argue that the common signatures imply that letter sequences, odor representations in *Drosophila*, and representations of animal categories in humans are underlain by a common representation. There is thus a similar logical impasse (known more commonly as the fallacy of "reverse inference" in, for example, the neuroimaging literature; Poldrack, 2006) when attempting to argue that qualitatively similar numerical ratio and distance effects demonstrate a common representation for symbolic numbers and nonsymbolic magnitudes.

Rather than relying on such indirect inference, Lyons, Ansari, and Beilock (2012) directly examined the behavioral implications of mixing numbers presented in symbolic and nonsymbolic formats. Specifically, adult participants were asked to directly compare a symbolic number with a nonsymbolic magnitude (an array of dots presented too quickly to count) and determine which represented the greater quantity. If the two visual formats essentially point to the same underlying representation, then performance should be little different than when comparing numbers in the same format (e.g., deciding which of two dot arrays contains more dots). Instead, results showed a very large cost of switching between formats (in particular, significantly longer response times). Note that the cost cannot simply be attributed to switching between two different types of visual stimuli, as no such cost was seen when switching between two symbolic formats: printed number words and Arabic numerals. In other words, results showed that integrating information across symbolic numbers and nonsymbolic magnitudes is much less efficient than would be expected if the two pointed to the same underlying representation. This indicates that symbolic and nonsymbolic numbers may in fact be represented more differently than had previously been assumed. It is important to note that this study was conducted with adults, which leaves open the possibility that young children's symbolic and nonsymbolic number representations are more closely linked (with the dissociation emerging slowly over the course of ensuing development). On the other hand, we noted earlier that the evidence for a link between the two formats in kindergarteners and preschoolers is mixed, at best. In sum, symbolic and nonsymbolic numbers are probably less directly associated with one another than one would expect if the former were bootstrapped directly from the latter; however, the developmental processes remain only partially understood at present.

Third, evidence for common neural processing of symbolic and nonsymbolic numbers appears to be far less convincing upon further examination. Perhaps one of the most influential studies showed cross-format fMRI adaptation (Piazza et al., 2007). Participants repeatedly saw a number presented in one format (e.g., the symbolic number "50"); then they saw a number in the other format (in this example, an array of dots). Sometimes the number in the new format would be the same magnitude (50 dots) and sometimes the number would change (e.g., 20 dots). The authors showed that activity in the parietal cortex was greater when the number changed than when it did not, indicating some degree of cross-format coding in this brain area. On the other hand, Cohen Kadosh et al. (2011) subsequently demonstrated that parietal brain areas are far more sensitive to changes in *format* than to changes in number. That is, the brain is perhaps more keenly tuned to the differences in numerical format than to their similarities.

The logical impasse of reverse inference (mentioned earlier in the context of distance effects) applies to much of the neuroimaging evidence as well. In other words, simply showing that two tasks coactivate the same brain area does not imply that the processing utilized in that area is the same for both tasks because the same brain area may subserve many different functions depending on the exact pattern of activity seen in that area (see, e.g., Anderson, Kinnison, & Pessoa, 2013; Anderson and Penner-Wilger, 2013; Poldrack, 2006). In response, several papers have looked not at how much a particular brain area is activated, but instead at the spatial patterns of activity within a brain region.[2] To date, four papers have directly assessed the notion of common distributed patterns of brain activity across symbolic and nonsymbolic number processing (Bulthé, De Smedt, & Op de Beeck, 2014; Damarla & Just, 2013; Eger et al., 2009; Lyons et al., in press). Bulthé et al. (2014) found no evidence at any of the spatial scales they examined for similar distributed representations across symbolic and nonsymbolic numbers. They also failed to find evidence that successful classification of different numbers in one format was capable of generalizing to the other format. Damarla and Just's (2013) results echoed those of Bulthé and colleagues. Lyons et al. (in press) also replicated Bulthé et al.'s central result: no evidence was found to indicate that the distributed pattern of neural activity for a given symbolic number—e.g., "6"—is related to the pattern of activity seen for the same number when presented nonsymbolically—six dots. Furthermore, Lyons et al. (in press) also showed that even the underlying representation structures—how the patterns of activity for different numbers relate to one another—are qualitatively different for symbolic and nonsymbolic numbers. Indeed, only one of the studies (Eger et al., 2009) found positive evidence indicating that spatial patterns of neural activity for symbolic and nonsymbolic numbers bear any relation to one another—and even in that case, decoding was unidirectional and only slightly above chance: 57% accuracy, where chance was 50%. Therefore, more fine-grained analysis of the underlying patterns of neural data provides scant evidence for the idea that there are similar representations in the brain for symbolic and nonsymbolic numbers.

This may be partially explained by recent evidence indicating that the neural overlap seen for symbolic and nonsymbolic numbers depends on task demands. While number comparison tasks (which of two items is numerically greater) showed overlap in brain activity in a canonical number region

[2] The specific spatial distribution of an activation pattern over multiple units (voxels) is by definition more specific than the activity of a single node or region. In theory, though the danger of reverse inference remains to some extent, it should thus be mitigated when considering spatial patterns of activity because these are less likely to be shared across highly disparate brain functions.

(the intraparietal sulcus), this was not true for brain activity during symbolic and nonsymbolic numerical *ordering* tasks (are items in numerical order; Lyons & Beilock, 2013). This result calls into question whether neural overlap between symbolic and nonsymbolic number processing indicates a fundamentally common representation or a more transient effect that is more reflective of task demands than basic underlying representation.

Thus, recent evidence has begun to erode a strong view of the notion that the meanings of number symbols are grounded in a direct reference to their nonsymbolic counterparts. An alternative explanation is that number symbols are initially linked exclusively via the exact, nonsymbolic quantities within the subitizing range (≤ 4)[3]—that is, numbers less than or equal to four may be mapped onto corresponding nonsymbolic magnitudes, but those outside this range are largely distinct from their nonsymbolic counterparts (Carey, 2004).

Le Corre and Carey (2007) provided evidence consistent with the view that an initial understanding of symbolic numbers is tied to the subitizing system. A crucial step in the development of numerical understanding occurs when children grasp the "cardinality principle"—that counting to any number yields the number in the set, as indexed by the last number said, and that this principle can be extended, theoretically, to any number. The authors showed that 3–4-year-old children were able to map number words onto arrays of objects (essentially name the number of objects) within the subitizing range prior to acquiring an understanding of the cardinality principle. However, children at this age failed to consistently map corresponding number words onto sets containing more than four items until several months *after* acquiring the cardinality principle. This finding indicates that the ability to map number symbols (number words in this case) onto nonsymbolic magnitudes can occur prior to acquiring the cardinality principle, but only for numbers within the subitizing range. Le Corre and Carey (2007) interpreted this as evidence that symbol-magnitude mapping within the subitizing range is a critical precursor to grasping the numerical meanings of symbolic numbers more generally.

Evidence in adults is consistent with this view: Lyons et al. (2012, reviewed earlier) showed that the cost of mixing symbolic and nonsymbolic

[3] Subitizing refers to rapid, exact apprehension of the number of objects in a set without explicit counting (Dehaene & Cohen, 1994; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994), is relatively stable over development (Schleifer & Landerl, 2011), and is limited to a capacity of about three or four items (a limit that, at least in adults, is related to the general processing capacity limit for visual short-term memory; Luck & Vogel, 1997; Piazza, Fumarola, Chinello, & Melcher, 2011).

formats for small numbers (≤4) is substantially smaller than that found for large numbers (>4; Lyons et al., 2012), suggesting that subitizable symbolic and nonsymbolic numbers may retain a link even into adulthood.

How, then, do number symbols that refer to larger magnitudes—those beyond the subitizing range—acquire meaning? One idea is that the meanings of larger numbers are bootstrapped from an understanding of smaller number symbols (Carey, 2004). Just how this occurs remains up for debate (Ansari, 2008). Some researchers have suggested the grammatical structure of language plays a central role in understanding the representational structure of large symbolic numbers (Almoammer et al., 2013; Carey, 2004; Le Corre & Carey, 2007; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; Sullivan & Barner, 2014). Another possibility is that visuo-spatial processing is most crucial (e.g., Gunderson, Ramirez, Beilock, & Levine, 2012). In mapping numbers onto a visual-spatial number line, understanding that the spatial distance between integers remains constant on a linear scale helps children understand that the same is true for integers—even those that are uncountably large (Siegler & Ramani, 2008). Another, if related, view suggests that understanding magnitudes as ordered sequences allows us to reason about very large numbers with which one is unlikely to have much direct perceptual experience (e.g., one million; Lyons & Beilock, 2009, 2011). Crucially, these factors need not be mutually exclusive. They each operate from different formulations of a common assumption: that the meanings of larger number symbols are at best only loosely tied to their nonsymbolic counterparts (for a view that explicitly disagrees with this assumption; however, see Feigenson et al., 2013). On that front, a major unanswered question concerns the precise mechanism—linguistic, spatial, ordinal, nonsymbolic magnitudes, a combination thereof, or some yet-to-be discovered factor—that links larger number symbols to one another. As such, this question is now a central driver of research in the field of numerical cognition.

5. SUMMARY AND CONCLUSIONS

Numbers play a critical role in our everyday lives, and acquiring numerical and mathematical skills is one of the central goals of formal education across the globe. Over the past three decades, researchers from the fields of Cognitive Science, Psychology, and Neuroscience have investigated how numbers are represented and processed in the brain and mind. A particular focus of this line of research has been on better understanding the foundations upon which the development of numerical and mathematical skills rest. To do this, researchers have sought to understand how sets of items (numerical magnitudes) are processed and represented from infancy onward. A large body of recent evidence has converged to suggest that humans share with other species the ability to approximately represent nonsymbolic numerical magnitude (e.g., arrays of dots). This ability has also been found in very young infants. Furthermore, brain-imaging evidence suggests the involvement of the parietal cortex during numerical magnitude processing in monkeys, young babies, children, and adults.

In view of the large body of evidence supporting the theory that there exists both phylogenetic and ontogenetic continuity in the representation of nonsymbolic numerical magnitude, it has frequently been contended that this representation serves as the basis of acquisition of symbolic representations of numerical magnitude (e.g., number words and Arabic numerals) over the course of human development. Theories of the development of symbolic number processing (e.g., Dehaene, 2008) as well as training studies (e.g., Lyons & Ansari, 2009) and computational models (e.g., Verguts & Fias, 2004) are underpinned by the assumption that symbols, such as number words and digits acquire their meaning (i.e., become symbolic representations of numerical magnitude) by becoming connected to the approximate, nonsymbolic representations of numerical magnitude that can be found across species and can be detected early in human development. This theory is certainly compelling and intuitive. However, as this literature review demonstrates, the empirical studies that have examined its predictions have not provided robust evidence in support of a strong link between the nonsymbolic, approximate representation of numerical magnitude and number symbols. One approach to testing the predicted connection between nonsymbolic and symbolic representations of number has been to examine correlations between children's nonsymbolic number discrimination abilities and their symbolic numerical and mathematical skills. If nonsymbolic representations of numerical magnitude provide the scaffold upon which more complex, symbolic numerical skills are built, then individual differences in nonsymbolic numerical magnitude representations should predict variability in children's formal, symbolic numerical, and mathematical skills. The data summarized here, however, do not provide strong support for this prediction. Specifically, the correlations between nonsymbolic magnitude processing and measures of arithmetic achievement have been found to be mostly weak and have not been shown to explain unique variance over and above symbolic number processing skills. Some evidence even suggests that improvements in symbolic number processing precedes improvements in nonsymbolic processing. In addition to studies that have correlated nonsymbolic numerical magnitude processing measures with symbolic measures of numerical and mathematical achievement, researchers have examined the connection between symbolic and nonsymbolic representations in experimental studies using both behavioral and brain-imaging methods. As this review demonstrates, and similar to correlational studies, these investigations have not provided evidence in support of a strong connection between symbolic and nonsymbolic, approximate representations of numerical magnitude. If anything, the bulk of the evidence reviewed here indicates that the differences between symbolic and nonsymbolic numbers may well outweigh any similarities between the two formats.

In light of our review of this research, we suggest that the "Symbol-Grounding Problem" in the field of numerical cognition—that is how symbols acquire their meaning beyond associations with one another (Harnad, 1990)—has not been solved. The hypothesis that this process can be explained through the development of a strong connection between number symbols and the well-documented approximate system for the representation of numerical magnitude is not supported by available data from children and adults at both the cognitive and neural levels of analysis. Therefore, a major challenge for the field of numerical cognition will be to explore alternative solutions to the "Symbol-Grounding Problem," such as the notion that it is through the connection with exact, nonsymbolic representations of number in the subitizing range (1–4) that children learn the rules of the number sequence and that these rules are then generalized to larger numbers without requiring a direct connection to nonsymbolic representations of numerical magnitudes. How children acquire and learn to manipulate symbolic numbers outside this range is currently an active and important area of research.

A resolution to the "Symbol-Grounding Problem" will not only significantly improve our understanding of how children acquire sophisticated, symbolic representations of numerical magnitude that give them the potential to become the economists, engineers, and scientists of the future, but it will also have important educational implications. Our understanding of the processes by which children learn the meaning of number symbols will inform the best ways in which children will be assisted in this critical learning process by their teachers and caregivers.

REFERENCES

Almoammer, A., Sullivan, J., Donlan, C., Marušič, F., Žaucer, R., O'Donnell, T., et al. (2013). Grammatical morphology as a source of early number word meanings. *Proceedings of the National Academy of Sciences of the United States of America*, *110*(46), 18448–18453.

Anderson, M. L., Kinnison, J., & Pessoa, L. (2013). Describing functional diversity of brain regions and brain networks. *NeuroImage*, *73*, 50–58.

Anderson, M. L., & Penner-Wilger, M. (2013). Neural reuse in the evolution and development of the brain: Evidence for developmental homology? *Developmental Psychobiology*, *55*(1), 42–51.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews. Neuroscience*, *9*(4), 278–291.

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency? *Journal of Experimental Child Psychology*, *117*, 12–28.

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology*, *114*(3), 375–388.

Brankaer, C., Ghesquière, P., & De Smedt, B. (2014). Children's mapping between non-symbolic and symbolic numerical magnitudes and its association with timed and untimed tests of mathematics achievement. *PLoS One*, *9*(4), e93565.

Brannon, E. M., & Terrace, H. S. (1998). Ordering of the numerosities 1 to 9 by monkeys. *Science*, *282*(5389), 746–749.

Buckley, P. B., & Gillman, C. B. (1974). Comparisons of digits and dot patterns. *Journal of Experimental Psychology*, *103*(6), 1131–1136.

Bulthé, J., De Smedt, B., & Op de Beeck, H. P. (2014). Format-dependent representations of symbolic and nonsymbolic numbers in the human cortex as revealed by multi-voxel pattern analyses. *NeuroImage*, *87*, 311–322.

Bynner, J., & Parsons, S. (1997). *Does numeracy matter?* London: The Basic Skills Agency.

Cantlon, J. F. (2012). Math, monkeys and the developing brain. *Proceedings of the National Academy of Sciences of the United States of America*, *109*, 10725–10732.

Cantlon, J. F., & Brannon, E. M. (2006). Shared system for ordering small and large numbers in monkeys and humans. *Psychological Science*, *17*(5), 401–406.

Cantlon, J. F., Brannon, E. M., Carter, E. J., & Pelphrey, K. A. (2006). Functional imaging of numerical processing in adults and 4-y-old children. *PLoS Biology*, *4*(5), e125.

Carey, S. (2004). Bootstrapping and the origins of concepts. *Daedalus*, *133*(1), 59–68.

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PLoS One*, 7(4), e33832.

Chen, Q., & Li, J. (2014). Association between individual differences in nonsymbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, *148*, 163–172.

Cohen Kadosh, R., Bahrami, B., Walsh, V., Butterworth, B., Popescu, T., & Price, C. J. (2011). Specialization in the human brain: The case of numbers. *Frontiers in Human Neuroscience*, *5*, 62.

Damarla, S. R., & Just, M. A. (2013). Decoding the representation of numerical values from brain activation patterns. *Human Brain Mapping*, *34*(10), 2624–2634.

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*(2), 48–55.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S. (2008). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. In P. Haggard & Y. Rossetti (Eds.), *Sensorimotor foundations of higher cognition (attention and performance)* (pp. 527–574). New York: Oxford University Press.

Dehaene, S., & Cohen, L. (1994). Dissociable mechanisms of subitizing and counting: Neuropsychological evidence from simultanagnosic patients. *Journal of Experimental Psychology. Human Perception and Performance*, *20*(5), 958–975.

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron*, *56*(2), 384–398.

Desoete, A., Ceulemans, A., De Weerdt, F., & Pieters, S. (2012). Can we predict mathematical learning disabilities from symbolic and nonsymbolic comparison tasks in kindergarten? Findings from a longitudinal study. *British Journal of Educational Psychology*, *82*(1), 64–81.

Dewind, N. K., & Brannon, E. M. (2012). Malleability of the approximate number system: Effects of feedback and training. *Frontiers in Human Neuroscience*, *6*, 68.

Diester, I., & Nieder, A. (2007). Semantic associations between signs and numerical categories in the prefrontal cortex. *PLoS Biology*, *5*, e294.

Diester, I., & Nieder, A. (2010). Numerical values leave a semantic imprint on associated signs in monkeys. *Journal of Cognitive Neuroscience*, *22*(1), 174–183.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A., Klebanov, P., et al. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446.

Eger, E., Michel, V., Thirion, B., Amadon, A., Dehaene, S., & Kleinschmidt, A. (2009). Deciphering cortical number coding from human brain activity patterns. *Current Biology*, *19*(19), 1608–1615.

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, *123*, 53–72.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314.

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives*, 7(2), 74–79.

Fias, W., Lammertyn, J., Reynvoet, B., Dupont, P., & Orban, G. A. (2003). Parietal representation of symbolic and nonsymbolic magnitude. *Journal of Cognitive Neuroscience*, *15*(1), 47–56.

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science*, *16*(1), 136–148.

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, *4*(2), 59–65.

Gelman, R., & Gallistel, C. R. (2004). Language and the origin of numerical concepts. *Science*, *306*(5695), 441–443.

Gilbert, A. L., Regier, T., Kay, P., & Ivry, R. B. (2008). Support for lateralization of the Whorf effect beyond the realm of color discrimination. *Brain and Language*, *105*(2), 91–98.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Nonsymbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition*, *115*(3), 394–406.

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children's arithmetic development: It is number knowledge, not the approximate number sense, that counts. *Psychological Science*, *25*(3), 789–798.

Gray, S. A., & Reeve, R. A. (2014). Preschoolers' dot enumeration abilities are markers of their arithmetic competence. *PLoS One*, *9*(4), e94428.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology*, *48*(5), 1229–1241.

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences of the United States of America*, *109*(28), 11116–11120.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665–668.

Harnad, S. (1990). The symbol grounding problem. *Physica D*, *42*, 335–346.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology*, *103*(1), 17–29.

Hubbard, E. M., Diester, I., Cantlon, J. F., Ansari, D., van Opstal, F., & Troiani, V. (2008). The evolution of numerical cognition: From number neurons to linguistic quantifiers. *Journal of Neuroscience*, *28*(46), 11819–11824.

Hyde, D. C., Boas, D. A., Blair, C., & Carey, S. (2010). Near-infrared spectroscopy shows right parietal specialization for number in pre-verbal infants. *NeuroImage*, *53*(2), 647–652.

Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief nonsymbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition*, *131*(1), 92–107.

Izard, V., Dehaene-Lambertz, G., & Dehaene, S. (2008). Distinct cerebral pathways for object identity and number in human infants. *PLoS Biology*, *6*(2), e11.

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. M. (2013). Early numerical development and the role of nonsymbolic and symbolic skills. *Learning and Instruction*, *25*, 95–103.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*(2), 395–438.

Libertus, M. E., & Brannon, E. M. (2009). Behavioral and neural basis of number sense in infancy. *Current Directions in Psychological Science*, *18*(6), 346–351.

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science*, *14*(6), 1292–1300.

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability? *Learning and Individual Differences*, *125*, 126–133.

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica*, *141*(3), 373–379.

Lonnemanna, J., Linkersdörfera, J., Hasselhorna, M., & Lindberg, S. (2011). Symbolic and nonsymbolic distance effects in children and their connection with arithmetic skills. *Journal of Neurolinguistics*, *24*(5), 582–591.

Lourenco, S. F., Bonny, J. W., Fernandez, E. P., & Rao, S. (2012). Nonsymbolic number and cumulative area representations contribute shared and unique variance to symbolic math competence. *Proceedings of the National Academy of Sciences of the United States of America*, *109*(46), 18737–18742.

Luck, S. J., & Vogel, E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature*, *390*(6657), 279–281.

Lyons, I. M., & Ansari, D. (2009). The cerebral basis of mapping nonsymbolic numerical quantities onto abstract symbols: An fMRI training study. *Journal of Cognitive Neuroscience*, *21*(9), 1720–1735.

Lyons, I. M., Ansari, D., & Beilock, S. L. (2012). Symbolic estrangement: Evidence against a strong association between numerical symbols and the quantities they represent. *Journal of Experimental Psychology. General*, *141*(4), 635–641.

Lyons, I. M., Ansari, D., & Beilock, S. L. (in press). Qualitatively different coding of symbolic and nonsymbolic numbers in the human brain. *Human Brain Mapping*, http://dx.doi.org/10.1002/hbm.22641. [Epub ahead of print].

Lyons, I. M., & Beilock, S. L. (2009). Beyond quantity: Individual differences in working memory and the ordinal understanding of numerical symbols. *Cognition*, *113*(2), 189–204.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, *121*(2), 256–261.

Lyons, I. M., & Beilock, S. L. (2013). Ordinality and the nature of symbolic numbers. *Journal of Neuroscience*, *33*(43), 17052–17061.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, *17*(5), 714–726.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology. General*, *111*(1), 1–22.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011a). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development*, *82*(4), 1224–1237.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011b). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS One*, *6*(9), e23749.

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature*, *215*(2), 1519–1520.

Mussolin, C., Nys, J., Content, A., & Leybaert, J. (2014). Symbolic number abilities predict later approximate number system acuity in preschool children. *PLoS One*, *9*(3), e91839.

Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annual Review of Neuroscience*, *32*, 185–208.

Park, J., & Brannon, E. M. (2013). Training the approximate number system improves math proficiency. *Psychological Science*, *24*(10), 2013–2019.

Park, J., & Brannon, E. M. (2014). Improving math with number sense training: An investigation of its underlying mechanism. *Cognition*, *133*(1), 188–200.

Parnas, M., Lin, A. C., Huetteroth, W., & Miesenböck, G. (2013). Odor discrimination in Drosophila: From neural population codes to behavior. *Neuron*, *79*(5), 932–944.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., et al. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33–41.

Piazza, M., Fumarola, A., Chinello, A., & Melcher, D. (2011). Subitizing reflects visuo-spatial object individuation capacity. *Cognition*, *121*(1), 147–153.

Piazza, M., Pinel, P., Le Bihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron*, *53*(2), 293–305.

Poldrack, R. A. (2006). Can cognitive processes be inferred from neuroimaging data? *Trends in Cognitive Sciences*, *10*(2), 59–63.

Sarnecka, B. W., Kamenskaya, V. G., Yamana, Y., Ogura, T., & Yudovina, Y. B. (2007). From grammatical number to exact numbers: Early meanings of 'one', 'two', and 'three' in English, Russian, and Japanese. *Cognitive Psychology*, *55*(2), 136–168.

Sasanguie, D., Defever, E., Maertens, B., & Reynvoet, B. (2014). The approximate number system is not predictive for symbolic number processing in kindergarteners. *Quarterly Journal of Experimental Psychology*, *67*(2), 271–280.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number-space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology*, *114*(3), 418–431.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science*, *14*(2), 280–291.
Sekuler, R., & Mierkiewicz, D. (1977). Children's judgments of numerical inequality. *Child Development*, *630–633*.
Shuman, M., & Kanwisher, N. (2004). Numerical magnitude in the human parietal lobe; tests of representational generality and domain specificity. *Neuron*, *44*(3), 557–569.
Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, *11*(5), 655–661.
Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America*, *110*(45), 18116–18120.
Sullivan, J., & Barner, D. (2014). Inference and association in children's early numerical estimation. *Child Development*, *85*(4), 1740–1755.
Szűcs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2014). Cognitive components of a mathematical processing network in 9-year-old children. *Developmental Science*, *17*(4), 506–524.
Toll, S. W., & Van Luit, J. E. (2014). Explaining numeracy development in weak performing kindergartners. *Journal of Experimental Child Psychology*, *124*, 97–111.
Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review*, *101*(1), 80–102.
Van Opstal, F., Gevers, W., De Moor, W., & Verguts, T. (2008). Dissecting the symbolic distance effect: Comparison and priming effects in numerical and nonnumerical orders. *Psychonomic Bulletin & Review*, *15*(2), 419–425.
vanMarle, K., Chu, F. W., Li, Y., & Geary, D. C. (2014). Acuity of the approximate number system and preschoolers' quantitative development. *Developmental Science*, *17*(4), 492–505.
Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: A neural model. *Journal of Cognitive Neuroscience*, *16*(9), 1493–1504.
Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of "The Number Race": An adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions*, *2*, 20.
Xu, F., & Spelke, E. S. (2000). Larger number discrimination in 6-month-old infants. *Cognition*, *74*, B1–B11.