Math Anxiety and Arithmetic Learning: Evidence for Impaired Procedural Learning and Enhanced Retrieval Learning

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ABSTRACT

Previous research has shown that high math anxiety (HMA) detrimentally impacts math performance; however, limited work has examined how math anxiety may impact math *learning*. The present study drew on our understanding of disparate long-term learning and memory systems to provide a framework for how HMA potentially disrupts specific types of math learning. Adult participants completed unfamiliar multiplication trials (e.g., 219×4=?) in 2 sessions across consecutive days. Repeated Problems enabled retrieval arithmetic learning by repeating the same 4 problems a total of 72 times each (288 total trials). Unrepeated Problems enabled procedural arithmetic learning by repeating a consistent problem structure but without ever repeating a specific problem (288 total trials). HMAs showed impaired learning of Unrepeated Problems suggesting MA may have disrupted procedural math learning. Conversely, learning of Repeated Problems was accelerated in HMAs relative to LMAs, suggesting enhanced retrieval learning. We interpret these results within the context of effort-avoidance and well-established learning and memory systems, suggesting that HMAs enhance effort on declarative memory mediated retrieval learning possibly at the expense of efficiency gains in procedural memory mediated learning of computational procedures. This work also suggests that the mechanisms linking math anxiety with math performance may differ in important ways from how math anxiety impacts math learning. Further, this work highlights the potential value of considering how math anxiety interacts with multiple types of math learning.

Key Words

Arithmetic; Math Anxiety; Retrieval Learning; Procedural Learning; Declarative Memory; Procedural Memory

1. INTRODUCTION

We live in a world that increasingly relies on numerical and mathematical skills. Consequently, anxiety about numerical and mathematical situations may be an important challenge for individuals to overcome in cultivating and understanding mathematics. Math anxiety refers to negative and nervous feelings associated with anticipating or completing mathematical tasks (Ashcraft, 2002; Ramirez et al., 2018; Suárez-Pellicioni et al., 2015). Short-term, the impacts of high math anxiety (HMA) can be seen in poor performance on arithmetic problems, and lowered performance in math coursework relative to peers. Long term, HMA individuals (HMAs) may take fewer math classes and are less likely to choose a career that relies on quantitative or numerical skills (Ashcraft, 2002; Daker et al., 2021; Hembree, 1990).

Despite the prevalence of research on the impact of math anxiety on math performance, very little work has directly examined whether – and in what manner – math anxiety impacts math *learning* (Dowker et al., 2016). While an examination of math learning to the fullest extent is outside the scope of a single empirical paper, the current study examines a subset of math learning which involves both multiple cognitive mechanisms, while also allowing for careful experimental control: complex mental arithmetic.

In the context of math anxiety, it is important to distinguish between explanations or mechanisms underlying *performance* and those underlying *learning*. For example, prior work has firmly established that an important explanation for why math anxiety is associated with decrements in math performance is a temporary decrement in working memory resources (Friso-van de Bos et al., 2013; Ji & Guo, 2023; Raghubar et al., 2010; Ashcraft & Krause, 2007a). However, decrements in performance are not the same as decrements in learning. For instance, a musician might challenge themself by attempting to play a composition at a tempo that is just outside their current skill level. Doing so may lead to an immediate decrement in performance, but ultimately lead to greater learning (relative to continuing to play the piece at the same tempo; Ericsson, 2008). Even in cases where decrements in performance and learning are observed in tandem, the mechanism underlying poor performance may be different from the mechanism underlying poor learning. For instance, sleep deprivation negatively impacts both cognitive performance and learning, but the mechanisms underlying each are not always the same (e.g., Curcio et al., 2006; Killgore, 2010). For these reasons, it is important not to confuse current explanations of arithmetic performance decrements among highly math anxious individuals (HMAs) with explanations of how such individuals may or may not learn (or fail to learn) arithmetic relative to their low math anxious peers (LMAs). Readers should note this is not meant as a criticism of those explanations of performance decrements in math anxiety, as, to our knowledge, they were not intended to serve as explanations of learning.

With respect to arithmetic learning, prior work has differentiated learning math facts from learning procedural calculations (Ashcraft, 1992; Delazer, 2003; Dowker, 2023). Importantly, prior literature has proposed that different cognitive mechanisms may underscore *learning* for these different types of math (Dowker, 2023; Menon, 2016b). For instance, researchers have suggested the declarative memory (DM) system supports recall of math facts, while additional, skills-based cognitive systems may contribute to learning of procedural computations (Dowker, 2023; Menon, 2016b). As such, it is important to consider the possibility that math anxiety may impact different types of arithmetic learning in different ways.

In particular, we are interested in examining whether math anxiety predicts reductions in two types of arithmetic learning: direct retrieval of arithmetic facts, and efficiency gains in executing procedural computations. The former occurs when a person sees many repetitions of the same problem (Ashcraft, 1983; Zbrodoff & Logan, 1986). For instance, most numerate adults have encountered problems like 2×3 many times, and so solve them by directly retrieving the answer from memory (Ashcraft, 1992). Adults can of course learn to retrieve less familiar items, such as 8×319, if they are presented with many repetitions of this same problem – indeed this is the basis of quite a few lab-based studies of arithmetic learning (e.g., Zbrodoff & Logan, 1986; Compton & Logan, 1991; Delazer et al., 2003; Rickard et al., 2008; Grabner et al., 2009; Battista, 2013). In our examination, the first question we ask is whether math anxiety disrupts retrieval learning of new arithmetic facts.

Efficiency gains in executing procedural arithmetic computations occur primarily by practicing execution of those procedures (Imbo & Vandierendonck, 2008). Note that this can be differentiated from direct retrieval learning by including problems where the structure of the problem is held constant, but specific instances (i.e., specific combinations of numbers) are repeated infrequently or not at all (e.g., Zbrodoff & Logan, 1986; Compton & Logan, 1991; Delazer et al., 2003; Rickard et al., 2008; Grabner et al., 2009; Battista, 2013). Perhaps a familiar example is long division, where there is often a common sequence of calculation steps that is largely invariant to the specific numerical inputs. Repeated practice with a fixed set of calculation steps can lead to increased efficiency in executing them, even if specific problems are repeated infrequently or not at all (e.g., Delaney et al., 1998; Imbo & Vandierendonck, 2008; Rickard et al., 2008; Battista, 2013). Note that execution of calculations for procedural arithmetic problems does make use of executive functioning and working memory capacity to enable mental manipulation of numerical values (Dowker, 2023). Importantly, the critical mechanism by which *gains in efficiency* for procedural arithmetic computations occur remains unclear. Thus, the second question we address is whether

HMAs experience reduced gains in arithmetic computation efficiency after practicing complex arithmetic problems with a fixed structure.

Why might math anxiety impact one or both of these types of arithmetic learning? Broadly speaking, heightened anxiety is associated with a change in how attention is allocated – typically toward exogenous (especially threatening) stimuli, and away from endogenous goals and representations (Pizzie & Kraemer, 2017; Dusek et al., 1976; Mogg et al., 1990; Moriya & Tanno, 2009). Similar attentional biases are thought to occur for math anxious individuals, and this bias is thought to be a leading cause of the performance decrements seen amongst highly math anxious individuals (HMAs) when they are doing math (Beilock, 2008; Ashcraft, 2002; Ramirez et al., 2018; Li et al., 2023; Daker et al., 2023). But as noted above, performance and learning are not the same, so the question at present is whether disruption of endogenous, goal-oriented attentional processes due to math anxiety disrupts different facets of arithmetic *learning*.

With respect to learning for arithmetic fact-retrieval, endogenous attention is an important component of explicit memory formation in the declarative memory (DM) system (Forsberg et al., 2021; Madore et al., 2020), which plays a crucial role in arithmetic retrieval learning (Dowker, 2023; Menon, 2016a, 2016b; Cho et al., 2012; Delezar et al., 2019; Grabner et al., 2009; Qin et al., 2014). Thus, by disrupting DM-mediated mechanisms, math anxiety could lead to disruption of arithmetic retrieval learning.

With respect to learning for computational procedures, one possibility is that math anxiety negatively impacts math practice. Here it is important to distinguish between quantity and quality of practice. Math anxiety is thought to lead to math avoidance (Dew et al., 1984; Pizzie & Kramer, 2017; Daker et al., 2021), potentially leading to a reduction in practice quantity. In the current study, the amount of practice was equated across all participants, so we are instead interested in how math anxiety might impact practice *quality*. That is, even if a high and a low math anxious person practiced a given arithmetic procedure the same amount, would the HMA person still experience reduced efficiency gains? HMAs reduce their effort on more challenging math problems (Choe et al., 2019; Jenifer et al., 2022), and procedural computations are generally perceived as more effortful than direct retrieval strategies (Ashcraft, 1992; Imbo & Vandierendonck, 2008; De Smedt, 2016). Thus, one possibility is that reduced effort by HMAs on more challenging problems involving computational procedures could lead to reduced practice quality. Because lower input quality can compromise procedural learning in general (Gupta & Cohen, 2002, Miller & Shettleworth, 2007), this in turn may lead to reduced efficiency gains in arithmetic computations among HMAs. While this proposal is admittedly somewhat speculative, the bottom line is that it is important to test whether HMAs show reduced computational, as well as retrieval, learning in arithmetic.

1.3. Current Study

1.3.1 Hypotheses

We propose three hypotheses for how math anxiety may impact arithmetic learning: (1) Math anxiety impairs arithmetic fact-learning; (2) Math anxiety impairs efficiency gains in a fixed computational context (e.g., computing the same *type* of arithmetic problem, without repeating any one specific problem); (3) Math anxiety impairs both.

1.3.2. Approach

The present study examined how math anxiety predicts the initial stages of adult learning of relatively difficult, multi-digit multiplication problems. These problems were chosen to be challenging, and unlikely to have been previously memorized (e.g., 189×4=?), and participants were given a 10-second time-limit per problem to encourage the cultivation of more efficient computational procedures. Participants completed an intensive battery (over 600 total trials) of multiplication problems divided evenly over two sessions that occurred on two consecutive days. All problems were open-ended, requiring participants to provide the answer themselves, and feedback in the form of the correct answer was given after each trial. Crucially, problems were divided into *Repeated Problems*, and *Unrepeated Problems*. Repeated Problems comprised a set of four multiplications problems that were each repeated 72 times throughout the experiment, enabling participants to rely primarily on declarative memory to recall the answers to these problems. Unrepeated Problems were all the same computational class (three-digit × one-digit number), but specific problems were never repeated. This meant that direct retrieval of answers to specific to Unrepeated Problems was not possible, but efficiency gains in consistent sequences of computational steps were possible (and encouraged by the time-limit).

To systematically address the hypotheses noted above, we investigated four research questions addressing different aspects of the impact of math anxiety on arithmetic learning. *First*, we sought to establish whether different Problem Types (Unrepeated Problems vs. Repeated Problems) show differential learning trajectories. We did this by examining the overall learning trajectories for each Problem Type across the course of the experiment and sought to establish whether learning on Unrepeated and Repeated problems indeed reflected differential types of arithmetic learning. *Second*, we tested whether HMAs are capable of each type of arithmetic learning – namely, we tested for the presence of learning among HMAs on each type of problem. *Third*, it is possible that HMAs demonstrate evidence of learning for a given problem type, but learning trajectories nevertheless differ from their LMA counterparts. To that end, we tested whether learning trajectories differed as a function of Math Anxiety for one or both problem types. *Fourth*, we tested whether the potential differential impact of math anxiety on a given type of learning is evident in the short-term (within a single session) or emerges primarily beyond a single testing session (i.e., after a 24-hour period).

On a broader scale, this work has the potential to advance our understanding of whether, in what manner, and on what timescale, math anxiety impacts math *learning*. Our results may also provide a bridge between math anxiety, math learning and long-term memory literatures.

2. METHODS

2.1. Participants

Participants were recruited from the student population at Georgetown University (n=84) and other adults in the surrounding community (n=5). (Note that results did not meaningfully differ if the community participants were omitted.) Participants were first recruited to participate in an online study and subsequently invited to participate in two in-lab sessions on consecutive days. 89 participants completed both in-lab sessions. Of those, survey data from 1 was lost due to a technical error; 6 others were dropped from the analysis because insufficient responses did not allow for response-times to be computed. Note that exclusion due to insufficient responses was unrelated to math anxiety [r=.06, p=.60]. The final analytic N was thus 82 (58 female, mean age: 22.45yrs, range: 18-49yrs).

2.2. Procedure

The initial online study was part of a larger dataset comprising a battery of questionnaires and several online tasks collected via Qualtrics. Task-order was counterbalanced across participants. Of primary relevance here are the measures of math anxiety, general trait anxiety, and the basic demographics data. Participants were subsequently invited to participate in the in-lab portion of the study. The in-lab portion consisted of two testing sessions over two consecutive days. The in-lab portion involved completion of an intensive multiplication task (see below for details). Electrodermal activity was recorded throughout both sessions, though here we focus solely on behavioral indicators of learning and their relation to math anxiety. Each lab session took approximately 90 minutes on average to complete. All procedures were approved by the University Institutional Review Board, and participants provided written consent to be a part of the study.

2.2.1 Transparency and Openness

We report how we determined our sample size, all data exclusions, manipulations, and measures. This study's design and analyses were not preregistered. Analyses were conducted using SPSS software Version 29. All data have been made publicly available at the APA's Open Science Framework (OSF) repository and can be accessed at the following link: https://osf.io/zc9nf/?view_only=2ef79dd4cbe64421b9f6447daa6f80ff.

2.3. Measures

2.3.1. Math Anxiety

Math anxiety was measured using the 25-item shortened math anxiety rating scale (SMARS; Alexander & Martray, 1989). Ratings on this scale range between 0 and 100. The mean rating in the current dataset was 35.2, with a standard deviation of 17.3.

2.3.2. General Trait Anxiety

General trait anxiety was measured using the 20-item trait portion of the state-trait anxiety index (TAI; Spielberger et al., 1970). Ratings on this scale range from 20 to 80. The mean rating in the current dataset was 47.1, with a standard deviation of 4.4.

Figure 1

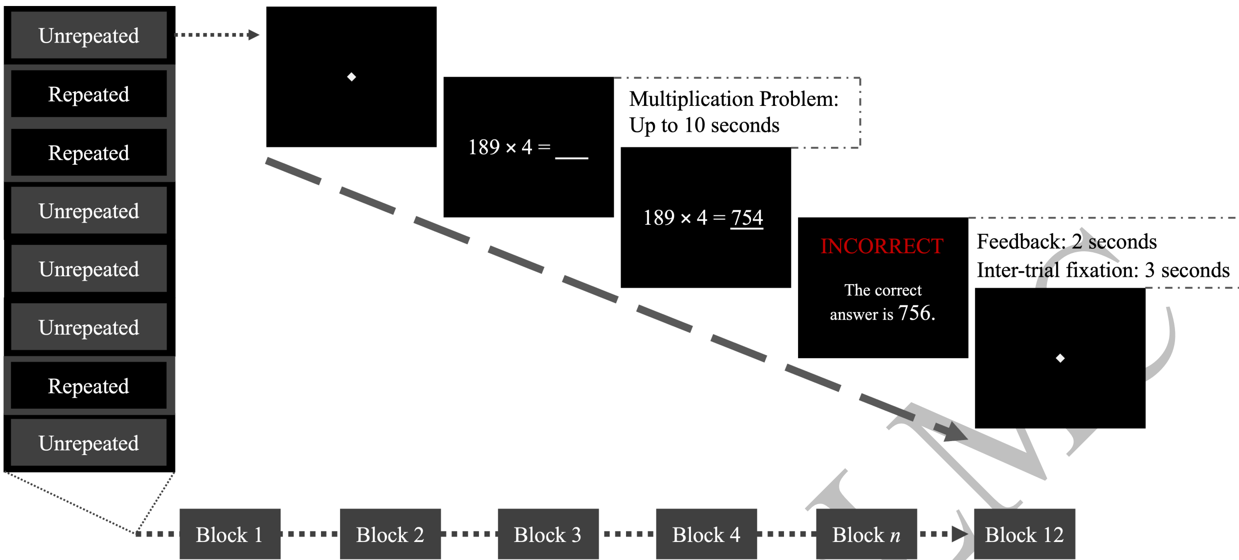

Figure 1 Notes. Figure 1 shows the experimental paradigm for a single session. On the left is a depiction of the type of randomization used for problem type presentation. Along the bottom we display the sectioning of the 12 Blocks within each session. Each Block contained 27 multiplication trials and lasted about 5 to 10 minutes. Participants were given short breaks in between Blocks. On the diagonal is a sample of the screen shown during each trial illustrating the initial fixation, the problem presentation and solving, feedback, and the subsequent return to fixation.

2.3.3. Multiplication Task

The multiplication task was presented via E-Prime 3.0 and displayed on a 1280×1024 standard Dell flat screen monitor. Multiplication problems were open-ended and required participants to type their answers via the number pad on a standard keyboard. Problems were designed to be moderately challenging and relatively unfamiliar. Problems consisted of two multiplicands presented horizontally in the form a × b = __. The left multiplicand (a) was always three-digits, ranging from 101-399. The second multiplicand (b) was a single-digit: 2, 3, 4, 6, 7, 8, 9. To equate the number of required key presses across problems, the solutions to all problems were three-digit integers (problems with products > 999 were excluded). The participant's answer appeared to the right of the equals sign as they typed their response. Stimuli were centered on the screen with problem text in 36pt Arial font. Each day, prior to the start of the main experiment, participants completed 5 practice problems. Practice problems were not repeated elsewhere in the experiment.

After a given trial appeared on the screen, participants had 10 seconds to complete their response. This 10-second response window was designed to create a certain amount of time-pressure and encourage learning, either via acquisition of more efficient strategies, such as memorization, or more efficient overall calculation processing. Responses were typed via the number pad and confirmed by pressing the Enter key (also on the number pad). Participants were allowed to correct their answers prior to pressing Enter, by using the Backspace key.

After each trial (either after pressing enter or after the 10-second time-limit expired), any typed response was scored, and participants received feedback. Feedback was either “Correct”, “Incorrect”, or “No Response Detected” (the latter occurred if participants failed to provide a response within the time limit). Font color for feedback was blue, red, or orange, respectively. Below the feedback text, was the following text in white: “The correct answer is:”, followed by the complete problem, including the correct answer (e.g., 189 × 4=756). The feedback screen was presented for 2 seconds, followed by an inter-trial fixation period of 3 seconds, after which the next trial began. The experiment was paused roughly every 5-10 minutes, and participants were given the option to take a short break.

Participants completed a total of 648 problems. These were divided evenly across two sessions that occurred on two consecutive days. Hence, participants completed 324 problems on Day 1 and 324 problems on Day 2. The 648 total problems were divided into 288 *Repeated Problems*, and 360 *Unrepeated Problems*. Note that the larger (three-digit) multiplicand in 72 of the 360 Unrepeated Problems involved a zero in the ones place (e.g., 280, 160, etc.). Due to concerns that this type of problem might afford qualitatively different types of strategies, they were omitted from analysis. Doing so also equalized the number of Repeated and Unrepeated Problems at 288 each. An equal number of each problem type was presented in each session (144 of each type on each day). Furthermore, problems were pseudo randomly presented so that, at any given point in a session, a participant would have completed roughly an equal number of Repeated and Unrepeated Problems (after excluding the 72 problems ending in a zero).

2.3.3.1. Repeated Problems

Repeated problems comprised the same four problems that were repeated throughout the experiment, including across both testing days. The four problems were 104 × 7, 142 × 3, 139 × 4, and 371 × 2. Each problem was repeated a total of 72 times across both sessions (36 times per session), which together comprised the 288 Repeated Problems participants saw in total. Note that these problems were chosen to be broadly representative of the types of problems (in terms of multiplicand place-value pairings) participants saw for the Unrepeated Problems.

Repeated Problems were designed so that learning – more specifically, accelerated learning beyond what is seen for Unrepeated Problems – on these problems would most likely be driven by direct memory retrieval. Feedback in the form of the correct answer was provided after each problem. Thus, by repeating a problem, one has the opportunity to accelerate learning by directly recalling the answer on subsequent problems – be that the answer one successfully produced oneself, or the answer provided via feedback (or both). Repeated Problems were randomly interspersed among a large number of Unrepeated Problems, meaning that participants could not simply memorize a single response routine.

2.3.3.2. Unrepeated Problems

Unrepeated Problems were never repeated throughout the experiment. Thus, all 360 Unrepeated Problems (288 of which were analyzed here) were presented exactly once. Unrepeated Problems were divided into two equal sets, and which set was presented on which Day 1 or Day 2 was counterbalanced across participants.

Unrepeated Problems were designed so that improved performance on this class of problems would most likely be driven by efficiency gains in computational procedures. No problem in this set was ever repeated, so participants could not rely on memorization of specific exemplars. However, the structure of the problem was held constant, allowing for consistent deployment of a small set of calculation procedures. Taken together, this meant that, if learning performance improvements over time were to occur for these problems, it would likely be due to improvements in the efficiency of the execution of that set of calculation procedures.

Note that, due to a technical error, the same set of Unrepeated Problems was accidentally presented on both days to 3 participants. Hence, those 3 participants in effect saw only 180 unique Unrepeated Problems which were presented exactly twice – once on each day. Including or excluding these participants did not affect the results. Two repetitions, when compared with 72 total repetitions of each Repeated Problem, is an order of magnitude less. On the other hand, one might expect the repetition across testing days could bias the effect of Day on learning trajectories for Unrepeated Problems, but this effect was in fact unchanged whether the 3 participants were excluded or not. Hence, for the sake of retaining as much data as possible, these 3 participants are included in subsequent analyses.

2.4. Analysis Framework

2.4.1. Z-Scores and Speed-Accuracy Trade-Offs

Our two measures of performance were accuracy and response time (RT). However, our focus in this study was on learning, which we quantified as changes in these performance variables. In particular, we were interested in directly contrasting changes as a function of Problem Type and math anxiety, which requires one use the same base variable (e.g., accuracy vs. accuracy, RT vs. RT). The issue is that speed-accuracy trade-offs are expected to differ across the two types of problems. For Repeated Problems, participants are expected to reach ceiling performance in terms of accuracy relatively quickly, at which point subsequent gains in processing would be more likely to be reflected by changes in RT. For Unrepeated Problems, no such switch in relative importance is expected. Thus, comparing learning in terms of either RT or accuracy could lead to inflated or deflated estimates of relative learning, depending on the measure chosen. Moreover, for Repeated Problems, precisely when the potential switch from changes in accuracy to changes in RT occurs is likely to differ across participants, which has implications for associating learning with an individual differences factor like math anxiety.

To avoid introducing these confounds into our analyses, we used a combined variable approach, in which we computed a composite measure of RT and accuracy. This implicitly accounts for potential trade-offs in relative gains in each of the two variables, resulting in an index of overall learning that is more directly comparable across Repeated and Unrepeated conditions. Furthermore, this approach halves the number of statistical comparisons needed, and reduces the chance that one is 'cherry picking' the outcome measure that is most convenient for one's hypotheses. Finally, the approach also implicitly accounts for individual variation in speed-accuracy trade-offs that might arise from idiosyncratic strategy selection.

To combine RT and accuracy, we used a z-score approach. We avoided inverse efficiency (Townsend & Ashby, 1983) as this weights accuracy non-linearly; we also opted not to use the Combined Performance (CP) metric introduced by Lyons et al. (2014), as that approach is more optimal in forced-choice situations, whereas here responses were open-ended. To compute z-scores, trials for each of the Problem Types were binned across 6 Timepoints as noted in the section on Assessing Learning Trajectories above. For each participant, average RT and error-rate (ER, % incorrect) was calculated for each Problem Type at each Timepoint. Z-scores were then computed as $z_i = -mean\big(z(RT_i), z(ER_i)\big)$, where $z(RT_i) = \frac{RT_i - RT_M}{RT_s}$ and $z(ER_i) = \frac{ER_i - ER_M}{ER_s}$. The i subscript indicates the value for a given participant for a given problem type for a given Timepoint. The *M* subscript indicates the grand mean across all participants, Problem Types and Timepoints. The *s* subscript indicates overall standard deviation across all participants, Problem Types and Timepoints. We used error-rates instead of accuracy because the former is in conceptually the same direction as RT (a lower value indicates 'better' performance). *Z*-scores were multiplied by -1 to aid in interpretation, such that a *higher* z-score indicated *better* overall performance.

2.4.2. Invalid Trials and Triaging Participants

To incentivize performance improvements, trials had a 10-second timeout; however, on trials where a timeout occurred, RT cannot be calculated, and it is unclear what this means in terms of accuracy. Hence, these trials were excluded from analysis. Some participants performed overall very poorly on the task, which often manifested as a very high number of timeout trials. Hence, we sought to triage such participants using a 75/25 rule: to be included in the sample, a given participant needed at least 75% valid (non-time-out) trials overall *and* at least 25% valid trials in each cell (Type × Timepoint). Six participants failed to meet one or both criteria and were excluded from further analysis. Neither the total number of invalid trials nor the likelihood of being excluded was significantly related to math anxiety (ps > .05).

2.4.3. Assessing Learning Trajectories

To assess learning trajectories for each of the Problem Types (Questions 1-3), we divided trials so that each Day contains 3 Timepoints, for a total of 6 Timepoints across the experiment. Note that this approach also allowed us to test for short-term learning across 3 Timepoints within a Day vs consolidation learning between daily Timepoints (Question 4). Each Timepoint comprised 48 trials of each Problem Type (Repeated, Unrepeated), and included 12 instances of each specific problem within the Repeated Problem set. In this way, we aimed to balance the capacity of our metrics to reasonably test each study question by estimating performance on the different problem types at each Timepoint. We used within-subjects contrast-effects to evaluate overall learning *trajectories* across timepoints. Note that contrast effects fit differences across levels in a given factor (e.g., the 6 levels in the Timepoint factor) to a specific mathematical function. Standard effects, conversely, detect only simple differences between levels, regardless of overall pattern.

2.4.4. Modeling Math Anxiety

Math anxiety is traditionally treated as a continuous measure (Dowker et al., 2016), though there is considerable precedent for comparing separate 'high' and 'low' math anxiety groups (e.g., Supekar et al., 2015; Passolunghi et al., 2020; Schaeffer et al., 2021; Jenifer et al., 2022; see also Ashcraft et al., 2007b). The argument against using groups is that group cutoffs are arbitrary, and this division often reduces statistical power. On the other hand, comparing groups is often conceptually easier to communicate, visualize, and understand. Also, Question 2 in the present study tests for learning in those classified as high math anxiety, necessitating group classification.

In the current sample, we first checked whether the association between math anxiety and overall performance varied as a function of whether math anxiety was treated continuously or in a group-based manner (using a median split). The median SMARS score in the sample was 33.5. Those with SMARS scores ≥ 34 were classified as high in math anxiety (HMA, n=41). Those with SMARS scores ≤ 33 were classified as low in math anxiety (LMA, n=41). For Repeated Problems, modeling math anxiety continuously was slightly stronger (Continuous: r=-.30, Group: r=-.26); for Unrepeated Problems, modeling math anxiety group-wise was slightly stronger (Continuous: r=-.33, Group: r=-.36); the two approaches were nearly identical with respect to overall performance (Continuous r=-.34, Group: r=-.34) (all ps<.05). Furthermore, within each group, we did not see a consistent association between continuous math anxiety and performance (LMA group: r=+.06, HMA group: r=-.24). Together, these preliminary results suggest (1) the statistical benefit of modeling MA continuously was not present in the current dataset, and (2) it may even be the case that there were qualitative differences between high and low math anxiety groups, making continuous treatment of MA potentially problematic for the current dataset. There is precedent for such qualitative differences elsewhere in the literature (e.g., Lyons & Beilock, 2012). Regardless, given that the potential benefits of modeling MA continuously in the present dataset did not seem to outweigh the benefits of modeling it group-wise, we opted to model MA in terms of high and low groups. Perhaps unsurprisingly given the above, none of the main conclusions of the paper are substantially altered if one were to choose to model MA continuously.

2.4.5. Accounting for General Anxiety

To establish specificity of math anxiety effects, it is customary to control for general anxiety, which we measured here using the trait portion (TAI) of the STAI. In the current dataset, we found no significant relation between math anxiety and general anxiety (r=.16, p=.150) or between general anxiety and overall performance (r=.17, p=.123). Further, general anxiety was not associated with any learning effects (all ps>.05; to estimate this, the ANOVAs from the results section were re-run substituting general anxiety for math anxiety, and by checking relevant interaction terms). Finally, results remained unchanged even after adjusting for general anxiety. Hence, in the current dataset, it does not seem necessary to control for general anxiety to establish specificity of math anxiety effects, and so for the sake of model simplicity, the main analyses omit trait anxiety as an additional factor.

3. RESULTS

3.1. Question 1: Do the different Problem Types show differential learning trajectories?

The first goal of the study was to establish whether in this paradigm, we see evidence of a reliance on differential learning and memory mechanisms for each Problem Type. We tested for learning on Unrepeated Problems (evidence for learning of computational procedures), and for learning on Repeated Problems (evidence for learning of direct retrieval). Figure 2 shows mean performance at each Timepoint for the two Problem Types. As this first research question does not concern math anxiety, the math anxiety group variable was omitted, and we consider all subjects together.

Note that the first timepoint in Figure 2 makes it appear as though there was a pre-existing difference between Repeated and Unrepeated Problems. For statistical estimation purposes Timepoint 1 in fact includes the first 48 trials of each problem type (in particular, 12 repetitions of each of the repeated problems), so some degree of learning may have already occurred within Timepoint 1. Thus, if we isolated accuracy on the very first instance that participants saw each of the four (soon-to-be) Repeated Problems, as well as accuracy on the first four Unrepeated Problems, there was no difference in accuracy between the problem types [Repeated: M=56%, *se*=2.97%; Unrepeated: *M*=55%, *se*=2.92%; $t(81)= -0.21$, $p=.84$]. Recall that responses were open-ended (i.e., not verification), so the low accuracy rates on these first four problems are still well above chance. More to the point, there was no reliable difference between Repeated and Unrepeated problems, at least at the very outset of the experiment. We next turn to examining variation in learning trajectories.

3.1.1. Log versus Linear Learning Trajectories

As illustrated in Figure 2, learning trajectories for both Problem Types was better fit by a log as opposed to a linear function of Timepoint at the group average level. A more quantitative approach confirmed this by using a one-way, within-subjects analyses of variance (ANOVA), and by examining the within-subjects contrast-effect, which provided the best estimate of overall learning trajectories, across the 6 Timepoints. We ran two ANOVAs for each Problem Type, modeling the contrast-effect of Timepoint either linearly and or a natural log function. The contrast-effect F-statistics for Timepoint from those models were as follows: Repeated Problems (Linear): $F=608.55$, Repeated Problems (Log): $F=630.40$; Unrepeated Problems (Linear): $F=159.84$, Unrepeated Problems (Log): $F=215.69$ (all *df*s: 1, 81; all *p*s<.001). For both Problem Types, the contrast effect of Timepoint was better fit by a log function as indicated by higher F-statistics.

Also of interest is the difference between learning trajectories on the two Problem Types. We checked whether the contrast effect for the interaction between Problem Type and Timepoint was better fit by a log, relative to a linear function. The contrast effect for the interaction term showed a better fit when Timepoint was modeled in a log (F=237.03) vs a linear manner (F=224.69). In sum, overall learning trajectories for both Repeated and Unrepeated Problems, as well as the difference between these two trajectories, were all better fit by a log function. We therefore model Timepoint as ln(Timepoint) moving forward.

3.1.2. Final Model

The final model for this section was 2(Type: Repeated, Unrepeated) × 6[Timepoint: *ln*(1-6)]. Within subjects' contrast effects are reported in-text, while full ANOVA results are in Appendix A. The main effect of Type was significant [$F(1,81)=875.04$, p=4E-45, $d=0.96$], such that Repeated performance was overall better than Unrepeated performance. The log contrast effect of Timepoint was significant [*ln*(Timepoint): $F(1,81)=707.01$, p=9E-42, $d=0.95$]. Substantial learning thus occurred overall. However, learning trajectories for the two different problem types were not equal, as evidenced by a significant interaction contrast-effect [Type × *ln*(Timepoint): $F(1,81)=237.03$, p=9E-26, $d=0.86$]. Significant learning was observed for Unrepeated Problems [*ln*(Timepoint): $F(1,81)=215.69$, p=2E-24, $d=0.85$], but learning was significantly accelerated for Repeated Problems [*ln*(Timepoint): $F(1,81)=630.39$, p=6E-40, $d=0.94$].

We thus show differential learning trajectories for the two problem types. We also find evidence that these different trajectories may be underlain by separate learning mechanisms: While overall performance across Problem Types was highly correlated [$r(80)=.679$, p=2E-12], *learning* trajectories for the two Problem Types were not [$r(80)=.133$, $p=.234$]. Furthermore, Unrepeated Problems showed evidence of learning in a context where problem structure was held constant, but without any specific problem ever being repeated. Repeated problems showed significantly accelerated learning in a context where a small subset of items was repeated. Taking the above together, it seems reasonable to conclude that learning on Unrepeated Problems and Repeated Problems were underlain by distinct memory mechanisms (We would of course not rule out some overlapping contribution of the two memory systems, though the lack of correlation between learning trajectories does speak against this to a degree.) In the next sections, we turn to the question of whether and how math anxiety potentially impacts math learning in each of these memory systems.

Figure 2

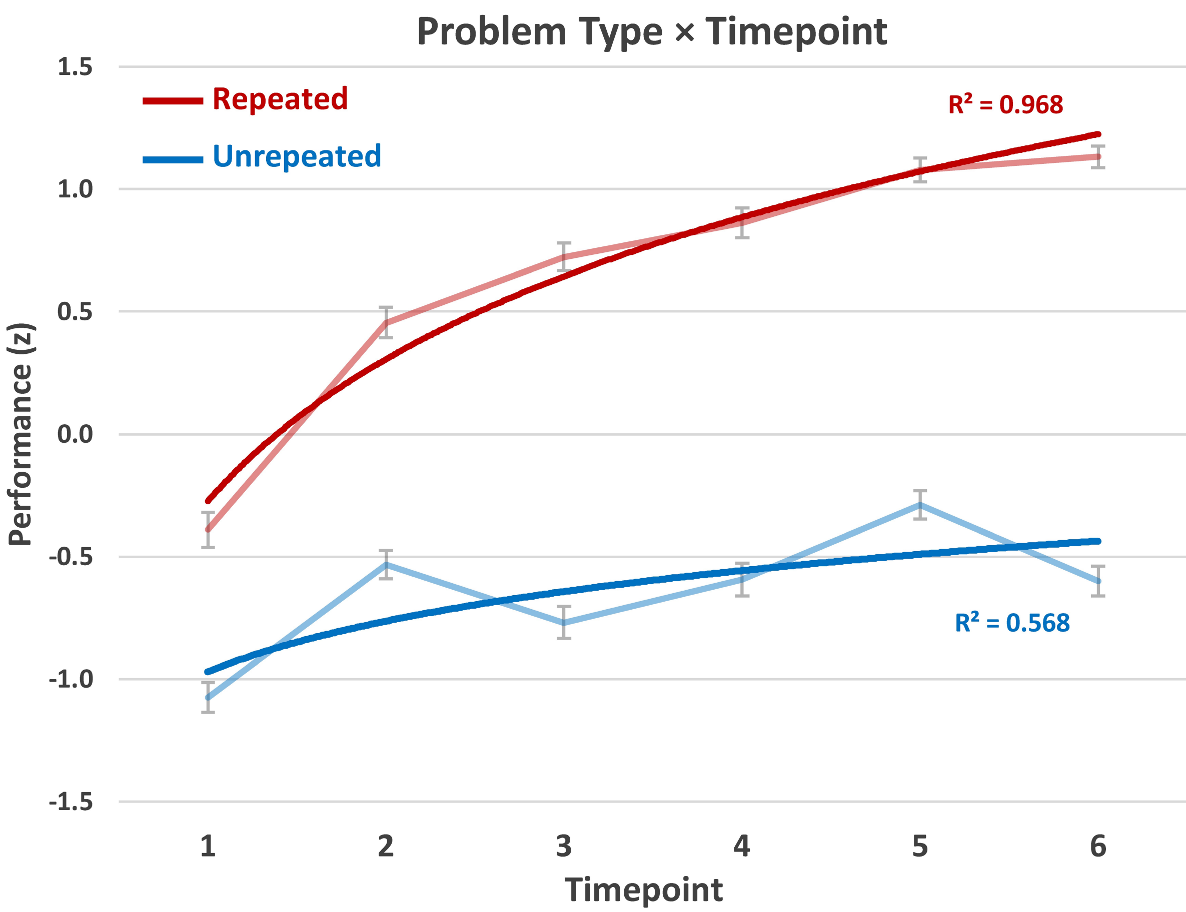

Figure 2 Caption: Figure 2 shows performance as a function of Problem Type and Timepoint. Performance is shown as z-scores. A higher z-score indicates better performance. Repeated Problems are in red; Unrepeated Problems are in blue. Learning is operationalized as consistent changes in performance and analyzed as log-contrast effects. To represent log-contrast effects, bold lines show fitted log functions: *ln*(Timepoint), along with R^2 values for the fitted lines. Shaded lines with error-bars (standard-errors) are actual timepoint means.

3.2. Question 2: Do HMAs have intact learning and memory mechanisms for arithmetic?

Here, we tested whether HMAs have fundamental deficits in one or both types of arithmetic learning (retrieval or procedural). Similar to Question 1, we tested for the existence of significant learning trajectories for each problem type, but here we focused exclusively on the HMA group (n=41). Within subjects' contrast effects are reported in-text, while full ANOVA results are in Appendix Table B-1 (LMA results are given in Table B-2 for completeness).

First, we checked for intact learning of computational procedures by testing for the presence of a significant learning trajectory among HMAs on Unrepeated Problems. We tested for the presence of a log-contrast effect across the 6 Timepoints. This contrast effect was indeed significant [*ln*(Timepoint): $F(1,40)=90.33$, p=8E-12, d=0.83], indicating learning mechanisms for arithmetic computational procedures remain intact among HMAs when learning math.

Second, we checked for intact retrieval learning by testing for the presence of an accelerated learning trajectory for Repeated relative to Unrepeated Problems. The log-contrast effect for Repeated Problems among HMAs was significant [*ln*(Timepoint): $F(1,40)=345.11$, p=2E-21, d=0.95]; moreover, this effect was significantly stronger than the effect for Unrepeated Problems above [Type × *ln*(Timepoint): $F(1,40)=199.08$, p=4E-17, d=0.91]. Arithmetic retrieval learning mechanisms thus appear to remain intact for HMAs when learning math.

3.3. Question 3: Do learning trajectories differ as a function of Math Anxiety?

While the previous section does not identify a fundamental deficit in retrieval or procedural arithmetic learning in HMAs, it is still possible that one or more mechanisms may be partially disrupted, preserved, or even enhanced among HMAs relative to their LMA counterparts. Here we test for these subtler differences by contrasting learning *trajectories* between HMAs and LMAs.

Figure 3 shows learning trajectories for each Problem Type and Math Anxiety Group. Within subjects' *contrast* effects (see Methods) are reported in-text, while full ANOVA results are in Appendices B and C.

The previous sections established a difference in learning trajectories as a function of Problem Type. Thus, in this section, we first tested whether this difference in learning trajectories in turn differed as a function of math anxiety. The three-way log-contrast effect was indeed significant [MA × Type × *ln*(Timepoint): $F(1,80)=11.92$, p=9E-04, $d=0.36$; Table C-1]. One way to understand this result is from the perspective of each Problem Type. While HMAs showed greater learning on Repeated Problems [MA × *ln*(Timepoint): $F(1,80)=5.36$, $p=.023$, $d=0.25$; Table C-2], LMAs showed marginally greater learning on Unrepeated Problems [MA × *ln*(Timepoint): $F(1,80)=3.67$, $p=.059$, $d=0.21$; Table C-3]. The differential learning trajectories in Figure 3 suggest different implications for MA-related arithmetic-learning deficits. There is an initial *performance* deficit for HMAs on both Problem Types. However, for Repeated Problems, due to accelerated learning, this performance deficit diminishes over time; for Unrepeated Problems, due to decelerated learning, the performance deficit increases with time.

Figure 3

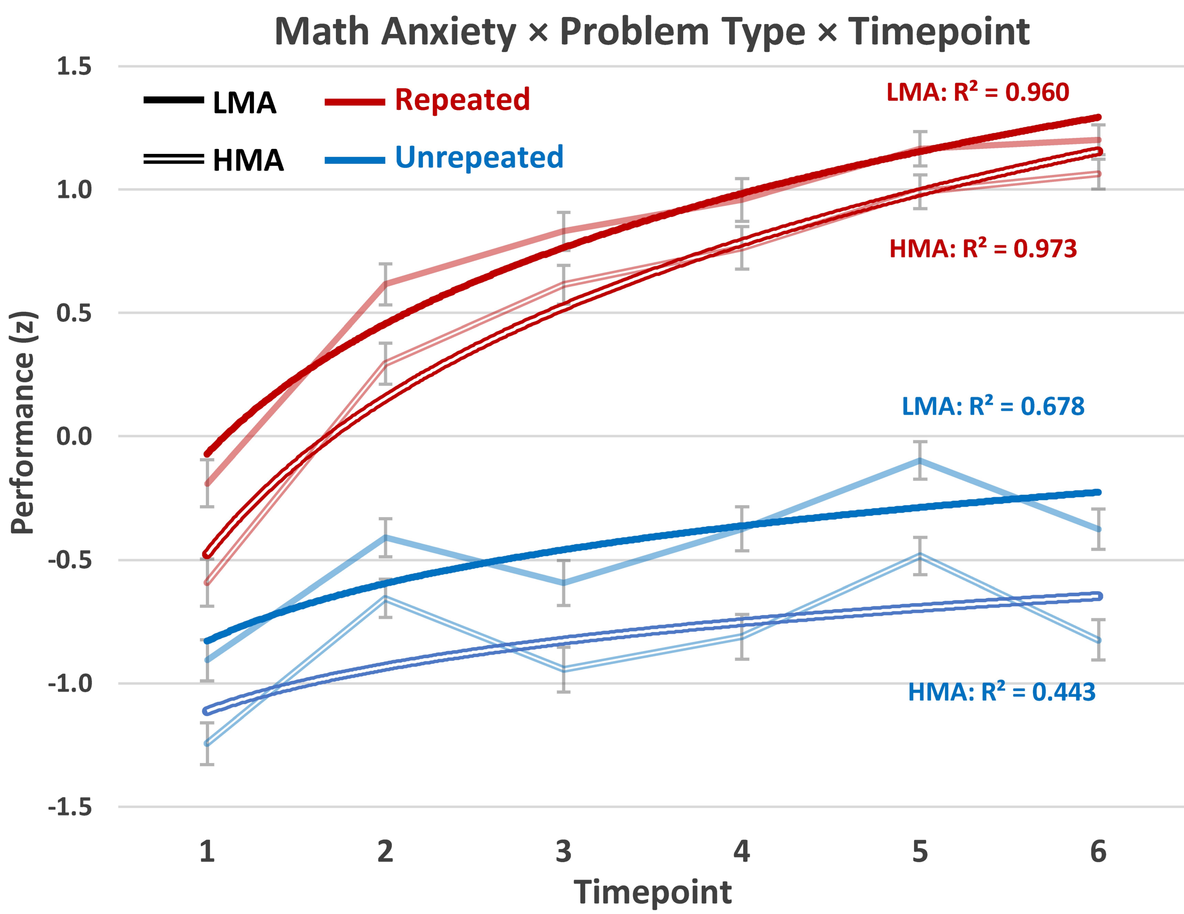

Figure 3 Caption: Figure 3 shows performance as a function of Problem Type and Timepoint, separated into low (LMA) and high math anxious (HMA) groups. Performance is shown as z-scores. A higher z-score indicates better overall performance. LMAs are shown with solid lines; HMAs are shown with hollow lines. Repeated Problems are in red; Unrepeated Problems are in blue. Learning is operationalized as consistent changes in performance and analyzed as log-contrast effects. Bold lines show fitted log-contrast-effect functions: *ln*(Timepoint), along with R^2 values for the fitted lines. Faded lines (both solid and hollow) with error-bars (standard-errors) are actual timepoint means.

Another way to think about the three-way contrast effect is that HMAs show accelerated learning on Repeated Problems (relative to Unrepeated Problems) to a greater extent than their LMA peers. This is reflected in the fact that HMAs showed a stronger Type × *ln*(Timepoint) effect [$F(1,40)=199.08$, p=4E-17, $d=0.91$; Table B-1] than did LMAs [$F(1,40)=82.99$, p=3E-11, $d=0.82$;

Table B-2]. Earlier, we interpreted accelerated learning on Repeated relative to Unrepeated problems as evidence of retrieval learning. Taken together with the above paragraph, these results suggest that MA disrupts procedural learning, but potentially enhances retrieval learning.

As a final note, after averaging over Problem Type, there was no significant difference in overall learning between HMAs and LMAs [MA × *ln*(Timepoint): $F(1,80)=0.74$, $p=.391$, $d=0.10$; Table C-1]. In other words, HMAs showed no evidence of differential learning relative LMAs when considering the arithmetic task as a whole. However, as we saw from above, this masks divergent learning trajectories as a function of Problem Type. This in turn highlights the importance of considering different types of learning and memory mechanisms when examining the impact of math anxiety on arithmetic learning. In sum, the evidence reviewed above *supports* Hypothesis (2): Math anxiety impairs learning in the form of enhanced arithmetic computational efficiency. Notably, the evidence *contradicts* Hypothesis (1): Math anxiety impairs learning of arithmetic facts (which thus also contradicts Hypothesis 3).

3.4. Question 4: Do differential Math Anxiety learning trajectories emerge in the short-term, or after 24 hours?

Having established the presence of differential learning trajectories by Problem Type between Math Anxiety groups, we looked to identify the time at which these differences emerge between groups. First, we tested whether trajectories diverge primarily within a given testing session. We operationalized this as learning *within* a testing Day. Second, we tested whether learning trajectories diverge in the (modestly) longer-term, possibly after memory consolidation processes have begun to occur. We operationalized this as learning *across* the two testing Days.

The current model was 2(MA: LMA, HMA) × 2(Type: Repeated, Unrepeated) × 2(Day: 1, 2) × 3(Timepoint: *ln*(1-3)). Note that in the current model there are only 3 levels to the factor Timepoint, because Timepoint here averages across Days (Timepoints 1 and 4 are averaged, as are 2 and 5, and 3 and 6).

Of primary interest here are two effects: (1) The MA × Type × *ln*(Timepoint) interaction contrast effect quantifies differential learning as a function of MA and Problem Type *across Timepoints (within a given testing session), ignoring the influence of Day*. (2) The MA × Type × Day interaction effect quantifies differential learning as a function of MA and Problem Type *across Days (between testing sessions), ignoring the influence of specific Timepoints within those Days*.

Results showed, at best, limited support for divergent learning trajectories within a given testing session [MA × Type × *ln*(Timepoint): $F(1,80)=2.30$, $p=.133$, $d=0.17$], but robust support for divergent learning trajectories across days [MA × Type × Day: $F(1,80)=10.66$, $p=.002$, $d=0.34$]. Note that the four-way interaction did not approach significance ($F<1$), indicating these two effects did not interact with one another. In sum, it appears that the differential Math Anxiety learning trajectories seen for the different Problem Types in Question 3 (Figure 3) did not fully emerge until the second of two testing days, which may suggest that the impact of Math Anxiety on retrieval and procedural learning may be driven by memory mechanisms that unfold over a timeframe that exceeds a single-session experiment.

4. DISCUSSION

Substantial work has demonstrated that high math anxiety is detrimental to math performance (Ashcraft & Ridley, 2005; Daker et al., 2021), which may give rise to the intuition that math anxiety is also detrimental to math learning. However, there is a lack of direct evidence for this intuition, and the limited evidence which does exist relies on longitudinal correlation studies identifying associations between math anxiety and outcome measures of previously learned math material. At the same time, there is a dearth of theoretical explanations in the literature outlining precisely *how* math anxiety might affect the process of math learning specifically. Taking an initial step towards filling this gap, the present study examined how math anxiety relates to two different types of arithmetic learning thought to be underlain by disparate learning mechanisms (Dowker, 2023; Menon, 2016b). Our results provide some of the first direct experimental evidence that math anxiety relates to impaired math learning – specifically, reduction of efficiency gains when practicing arithmetic calculation procedures. This result is broadly consistent with the notion that HMAs avoid effortful math, extending it to show that this avoidance may have negative implications for learning. Importantly, our results also show that math anxiety is *not* predictive of poor math learning across the board, as HMAs show preserved or even enhanced arithmetic fact-retrieval learning. This latter result underscores the need to consider how math anxiety impacts different types of math learning. In the discussion that follows, we provide a tentative interpretation of these results couched in literature on long-established learning and memory systems. The current work thus has the potential to provide a bridge-point between the math anxiety, math learning and long-term memory literatures.

4.1. Summary of Research Questions

Question 1. We operationalized the distinction between types of learning by examining how math anxiety interacted with learning of Repeated vs. Unrepeated Problems. Repeated Problems were designed to reflect retrieval learning, allowing participants to engage in rote memorization of the repeated arithmetic problems. Unrepeated Problems were designed to reflect gains in arithmetic calculation efficiency. To provide support for this operationalization of Problem Types (**Question 1**), we first sought to verify that different Problem Types (Repeated Problems vs. Unrepeated Problems) showed differential learning trajectories (regardless of math anxiety level). Indeed, they did (see Fig. 1), and while *performance* on the two problem types were highly related, *learning* trajectories were unrelated to one another. Specifically, we found significant learning for Unrepeated Problems, despite repetition only of problem structure (and not of individual items), providing evidence of efficiency gains in arithmetic calculation procedures. When problems were repeated (Repeated Problems), learning was significantly accelerated, providing evidence for the involvement of retrieval-based arithmetic learning.

Question 2. We next assessed whether HMAs' (retrieval and procedural) learning and memory mechanisms remained intact for math (**Question 2**). Specifically, we tested for the presence of significant learning trajectories in HMAs for each Problem Type. HMAs indeed demonstrated significant positive learning trajectories for both problem types, indicating that both retrieval and procedural learning mechanisms appear to remain intact for HMAs while learning math (Figure 3, hollow lines). However, demonstrating that neither type of arithmetic learning is completely compromised is not the same thing as demonstrating that they are not impaired (relative to LMAs).

Question 3. We thus directly compared HMA and LMA learning trajectories for each Problem Type (**Question 3**). HMAs showed an initial performance deficit compared to LMAs in both problem types (Timepoint 1, Figure 3); however, this deficit diminished over time for Repeated Problems, and increased over time for Unrepeated Problems (learning trajectories in Figure 3). This result, supported by the significant three-way interaction (MA × Type × *ln*(Timepoint); Table C-1), indicates that the impact of math anxiety on math learning depends on the type of learning involved. Overall, retrieval learning was accelerated, and this acceleration was significantly greater for HMAs (compare Type × *ln*(Timepoint) contrast effects in Tables B1 and B2). This greater acceleration was in turn driven by significantly accentuated retrieval learning for HMAs (Table C-2) and (marginally) attenuated procedural learning for HMAs (Table C-3). Put simply, HMAs were able to learn Repeated Problems (arithmetic retrieval) better than LMAs but learned Unrepeated Problems (arithmetic procedures) worse than LMAs. These results from Question 3 provide support for the second hypothesis put forth in the Introduction that high math anxiety may reduce quality of math practice, resulting in impaired efficiency gains on arithmetic calculation procedures. However, our results also argue directly against our first hypothesis that high math anxiety impairs learning of arithmetic facts. Consequently, results also refute our third hypothesis that math anxiety concurrently impairs retrieval and procedural arithmetic learning.

Question 4. Lastly, we sought to determine whether the differential effect of math anxiety on different types of arithmetic learning manifested in the short-term (within a single session) or after 24 hours (between two testing days; **Question 4**). Results indicated that differential trajectories primarily emerged between, as opposed to within, testing days, as evidenced by a stronger MA × Type × Day effect than an MA × Type × Time effect (Table D-1). Put simply, math anxiety's differential impact on retrieval and procedural arithmetic learning is most evident after a 24 hour, overnight, period. One potential driver of this effect may be the process of overnight consolidation, which is the neurocognitive process by which new memories are stabilized (Ullman & Lovelett, 2018; Marshall & Born, 2007). Consolidation is generally considered to improve learning in long-term memory systems (Mednick et al., 2011; Rasch and Born, 2013; Ullman, 2016). However, research indicates that there may be differential effects of overnight consolidation on different types of learning, with factors such as stress, sleep quality, and the content of learned materials affecting subsequent memory system function (Stamm et al., 2014; Diekelmann et al., 2009; Ullman & Lovelett, 2018). Consolidation processes are complex, and our results indicate that, particular to math learning, 24-hour spaced sessions which allow for overnight consolidation may produce differential effects on retrieval vs procedural arithmetic learning as a function of math anxiety. Future studies investigating the impact of math anxiety on math learning may need to examine learning over more than one session, and across multiple days to observe similar results.

4.2. Interpretation of Results in the Context of Effort and Long-Term Memory Systems

4.2.1. Long-Term Memory Systems and Arithmetic Learning

As noted in the Introduction, previous researchers have suggested that the two types of arithmetic learning considered here – retrieval vs procedural – may rely on disparate long-term learning and memory mechanisms (Dowker, 2023; Menon, 2016b). In particular, the declarative memory (DM) system may support arithmetic fact-retrieval, while a skills-based system – like procedural memory (PM) - may support procedural computations.

The DM system is optimally suited for acquisition of both arbitrary facts, but also for extraction of the types of semantic associations that are thought to underly much of arithmetic understanding (Ashcraft, 1983, 1982; Campbell, 2015). Moreover, brain structures that support DM have been implicated in arithmetic fact-retrieval learning (Cho et al., 2012; Delezar et al., 2019; Grabner et al., 2009; Qin et al., 2014; Menon, 2016b). It is thus not unreasonable to imagine that retrieval-based learning of Repeated Problems in the current context was at least partially mediated by DM learning mechanisms.

Conversely, the PM system is optimally suited for identifying and accelerating processing of high-frequency sequences (Ferbinteanu, 2019; Seger, 2006). Multi-step arithmetic in particular involves concatenation of discrete calculation steps, often in a specific sequential order (Ashcraft, 1992). Performance improvements in arithmetic (e.g., faster response times) can arise from increasing the efficiency of these specific calculation sequences (Compton & Logan, 1991; Imbo & Vandierendonck, 2008; Thevenot et al., 2007, 2020), even when participants are sometimes unable to identify such patterns (Rosenbaum, 2001; Wenger & Carlson, 1996; Seger, 2006; Menon, 2016b; Barrouillet & Thevenot, 2013). Indeed, there is mounting evidence that previously assumed hallmarks of arithmetic retrieval, such as size effects, may at least partially reflect efficiency gains in calculation procedures (Thevenot et al., 2007, 2020; Barrouillet & Thevenot, 2013). Work at the neural level has shown that PM brain regions are parametrically modulated by 'classic' arithmetic effects, including problem-size effects as well as memory interference effects (Tiberghien et al., 2019). Furthermore, Fias et al. (2021) found – in an alphabet arithmetic learning context – that it was primarily PM regions that showed activation curves consistent with learning and improvement of efficiency for calculation procedures (but not retrieval learning). In the current study, given that Unrepeated Problems comprised a highly consistent structure, but without repetition of specific problems, we suggest that learning – in the form of efficiency gains in executing computational procedures – on these problems was at least partially mediated by PM learning mechanisms.

In sum, prior theoretical and empirical work – at both the behavioral and neural level – supports the idea that the DM system contributes to arithmetic retrieval learning, and the PM system contributes to efficiency gains in executing arithmetic calculation procedures. In turn, though admittedly speculative, we suggest that learning on Repeated Problems may have been mediated by DM systems and that on Unrepeated Problems by PM systems.

4.2.2. Math Anxiety, Arithmetic Learning, and Memory Systems – The Role of Effort

In formulating our first hypothesis, we assumed that math anxiety would be a strictly disruptive force with respect to endogenous attention and would thus compromise retrieval learning. Our data show this assumption was incorrect. In light of our findings, we now offer an alternative interpretation of our results. Work by Choe et al. (2019) and Jenifer et al. (2022) has shown that high math anxiety leads to avoidance of high-effort math problems and math problem-solving strategies during math performance (though see also Thronsen et al., 2022). Here we suggest that this anxiety-related avoidance may lead HMAs to allocate resources toward reducing future effortful engagement during an arithmetic learning paradigm. Retrieval is comparatively less effortful than calculation (Ashcraft, 1982; Imbo & Vandierendonck, 2008), and – across the timescale of our experiment – retrieval *learning* may be relatively less effortful than procedural learning as DM-mediated learning is capable of operating over shorter-time scales (on the order of a few trials in adult humans; Ullman, 2016). Hence, in seeking the quickest route to reducing effortful arithmetic engagement, HMAs may have allocated greater resources to mastering retrieval of Repeated Problems via rapid DM-mediated learning mechanisms.

Conversely, while PM-mediated efficiency gains can ultimately lead to reduced effort, these gains generally come only after many repetitions (Ullman, 2016). Hence in the current experiment, there was no simple path for HMAs to reduce effort on Unrepeated Problems, leading HMAs to potentially allocate fewer resources to them. This in turn may have degraded the quality of practice on these items, and because PM-mediated learning relies not just on quantity but also quality of practice (Gupta & Cohen, 2002), degraded practice may have reduced the efficacy of PM-mediated learning mechanisms. For HMAs, accelerated retrieval learning on Repeated Problems and decelerated learning on Unrepeated problems my thus have widened the gap in the amount of effort required to complete each class of problems, thereby creating an insalubrious feedback loop. Math anxiety is known to lead to undesirable feedback cycles (Gunderson et al., 2018; Ashcraft, 2019; Dowker, 2019), and the current results may thus be another such example: HMAs commit less effort to practicing Unrepeated Problems, which increases the disparity in effort needed to compute them relative to Repeated Problems, leading to still less effort on Unrepeated Problems, and so on.

While the interpretations provided in this section are admittedly speculative – and in the case of Repeated Problems also post-hoc – they are nevertheless grounded in multiple literatures. Our primary aim in presenting these interpretations is of course not to definitively claim they are correct, but to offer them as speculative but plausible hypotheses that may prove useful for future research across multiple literatures.

4.2.3. Working Memory

It is important to consider an alternative interpretation of our result showing impaired learning for HMAs on Unrepeated Problems – namely that the negative impact of MA on these problems operated primarily via well-known impairments of working memory (WM) among HMAs when doing math (Friso-van de Bos et al., 2013; Ji & Guo, 2023; Raghubar et al., 2010; Ashcraft & Krause, 2007a). Temporarily reduced WM among HMAs generally leads to lower *performance* for HMAs on more WM-demanding problems (e.g., Ashcraft & Krause, 2007a; Lyons & Beilock, 2012). The question is whether differences in WM demands on the two problem types would predict the HMA-related *learning* patterns observed here. To understand how WM might relate to learning in the current context, it is useful to turn to literature on different memory systems.

Ablation work with animals (Packard et al., 1989) and research in the domain of language acquisition (Ullman, 2020) indicate that WM engagement may in fact disrupt PM learning. In another example from category learning, higher WM capacity is associated with more rapid learning of explicit, rule-based categorical structures thought to be mediated by DM, and *decelerated* learning of implicit, integrative categorical structures thought to be mediated by PM (DeCaro et al., 2008). In the current experiment, when completing the arithmetic task, the group we expect to have greater WM resources is LMAs, due to WM being partially compromised in HMAs (Friso-van de Bos et al., 2013; Ji & Guo, 2023; Raghubar et al., 2010; Ashcraft & Krause, 2007a). Extrapolating from the work reviewed above, we would thus expect LMAs to show an advantage in arithmetic retrieval learning, and a disadvantage in arithmetic procedural learning – that is, accelerated learning (relative to HMAs) on Repeated Problems and decelerated learning on Unrepeated Problems. However, our results showed precisely the opposite.

In sum, while we certainly agree with prior work that a WM-based account can help explain why HMAs and LMAs *perform* on math tasks differently, it appears that, at least in the current context, it fails to account for how HMAs and LMAs *learn* (different types of) arithmetic differently. More broadly, we see this as an important example of how mechanisms that explain performance are not necessarily those that explain learning.

4.3. Implications and Methodological Considerations

One potential implication of these results is that the impact of math anxiety may be different for arithmetic performance vs arithmetic learning. Similarly, mental arithmetic relies on a highly varied set of distinct and overlapping cognitive processes (Campbell, 2005; Dowker, 2023), and improvements in those disparate processes appears to entail disparate learning mechanisms (Menon, 2016b; De Smedt, 2016; Dowker, 2023). Thus, how math anxiety impacts arithmetic learning also varies as a function of the relevant learning mechanism. This observation may have implications for teaching arithmetic. For instance, perhaps contrary to claims that rote retrieval-based approaches to arithmetic learning (e.g., 'drill and kill') are especially debilitating for HMAs (e.g., Boaler, 2015), our results suggest this may be one area of arithmetic learning where HMAs might in fact excel. Indeed, work on foundational arithmetic retrieval might present an opportunity to build confidence in HMAs, while also cementing at least a subset of crucial arithmetic fluency skills. On the other hand, over-reliance on memorization-based strategies might come at the expense of fluency with executing more generalizable computation algorithms, which might put HMAs at a disadvantage when rote retrieval of certain items is not applicable.

Our results also point to two methodological implications. First, they highlight the importance of considering different types of learning, even in the limited context of mental arithmetic learning. Differential learning trajectories between math anxiety groups only emerged when we isolated learning within each Problem Type (Repeated vs. Unrepeated). Had we aggregated the total learning trajectories across Problem Types, results would have suggested a net zero effect of math anxiety on learning (the MA × *ln*(Timepoint) effect did not approach significance in Table C-1).

Second, differential learning trajectories between Problem Types emerged only when examined across two days. These results indicate that 24-hour spaced practice, possibly allowing for overnight consolidation, may be necessary to observe and unpack the various ways in which math anxiety does and does not impact math learning. Future examinations of math learning may do well to take these methodological points into consideration, utilizing different types of math problems and being cognizant of choosing timescales within which learning is anticipated to occur.

4.4. Limitations

One important limitation of the current research is that we considered how math anxiety impacts only a very small subset of math learning. Further, while we examined two forms of arithmetic learning, we would certainly not claim to therefore have examined all possible types of arithmetic learning. Indeed, as our results clearly point to the conclusion that math anxiety impacts different types of arithmetic learning differently, this perhaps makes it all the more important for future work to examine how math anxiety impacts not just other aspects of arithmetic learning, but various types of math learning more broadly.

A second limitation is that while we assessed the effect of 24-hour spacing on arithmetic learning, our study did not examine longer-term learning, and we cannot say whether the patterns of learning seen in our results would extend across these longer timescales. Future work investigating different timescales of math learning would be especially important given that timescales greater than 24-hours are common in math education settings. It is also important to note that the retrieval and procedural arithmetic learning in the present study was experienced in tandem, with different problem types interspersed with one another throughout the math learning task. If one were to offer discrete arithmetic learning tasks, results may differ.

A third limitation is that one of our main assumptions – and thus our first hypothesis – was wrong. While we presented a potential interpretation of our results by learning on prior literature on long-term memory systems and how math anxiety impacts avoidance and effort allocation, this interpretation is admittedly speculative. Future work might put this interpretation to the test in (at least) two ways. First, future work might more explicitly test whether there is a link between learning on Repeated Problems and DM mechanisms, and between learning on Unrepeated Problems and PM mechanisms. Second, neural or psychophysiological correlates of effortful processing might be deployed to more expressly test whether HMAs expend cognitive effort differently than HMAs in arithmetic learning.

Despite these shortcomings, we hope the current study provides a reasonable initial step toward expanding research on the intersection between math anxiety, arithmetic learning and long-term memory systems.

4.5. Conclusions

Broadly, our results suggest that HMAs redirect cognitive resources away from practicing computationally intensive arithmetic calculation procedures and toward rapid rote-memorization of arithmetic facts. Further, the current work highlights the importance of distinguishing between performance and learning when considering the implications of math anxiety. Perhaps of particular interest, our results also show that math anxiety is *not* predictive of poor math learning across the board; even in the limited context of arithmetic learning, HMAs showed preserved or even enhanced arithmetic fact-retrieval learning. This latter result underscores the need to consider how math anxiety impacts different types of math learning, and it may provide a leverage point for initiating confidence-building interventions with math anxious individuals. On the other hand, the tendency to neglect effortful practice of arithmetic calculation procedures may lead to an undesirable feedback loop, with ever increasing reliance on inflexible retrieval-based strategies among HMAs. While we believe this work provides insight into how math anxiety impacts different types of arithmetic learning, we believe it is merely a starting point. Substantial future work is needed to fully unpack the myriad ways in which math anxiety may interact with the different forms of math learning.

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APPENDIX A

Table A-1

2(Type: Repeated, Unrepeated) × 6[Timepoint: ln(1-6)]

	Dfn	Dfd	F	Cohen's D	p
Within Subjects Effects					
Type	1	81	875.04	0.96	3.54E-45
Timepoint	5	405	325.55	0.89	2.09E-139
Type x Timepoint	5	405	111.19	0.76	9.95E-74
Within Subjects Contrasts					
Timepoint	1	81	707.01	0.95	8.98E-42
Type x Timepoint	1	81	237.03	0.86	8.96E-26

Table A-1 Notes. Table A-1 shows the results of the final model from Research Question 1. Note that contrast effects are the natural log of Timepoint [ln(1-6)]. As Type has only two levels, the contrast effect of Type is identical to the Within-Subjects effect of Type.

APPENDIX B

Table B-1

HMA: 2(Type: Repeated, Unrepeated) × 6[Timepoint: ln(1-6)]

	Dfn	Dfd	F	Cohen's D	p
Within Subjects Effects					
Type	1	40	401.61	0.954	1.81E-22
Timepoint	5	200	142.31	0.884	7.09E-64
Type x Timepoint	5	200	87.04	0.828	2.85E-48
Within Subjects Contrasts					
Timepoint	1	40	328.54	0.944	6.82E-21
Type x Timepoint	1	40	199.08	0.913	4.04E-17

Table B-1 Notes. Table B-1 shows the results of the HMA model from Research Question 2. Note that contrast effects are the natural log of Timepoint [ln(1-6)]. As Type has only two levels, the contrast effect of Type is identical to the Within-Subjects effect of Type.

Table B-2

LMA: 2(Type: Repeated, Unrepeated) × 6[Timepoint: ln(1-6)]

	Dfn	Dfd	F	Cohen's D	p
Within Subjects Effects					
Type	1	40	511.56	0.96	2.10E-24
Timepoint	5	200	193.90	0.91	1.17E-74
Type x Timepoint	5	200	37.02	0.69	9.15E-27
Within Subjects Contrasts					
Timepoint	1	40	385.10	0.95	3.89E-22
Type x Timepoint	1	40	82.99	0.82	2.65E-11

Table B-2 Notes. Table B-2 shows the results of the LMA model from Research Question 3. Note that contrast effects are the natural log of Timepoint [ln(1-6)]. As Type has only two levels, the contrast effect of Type is identical to the Within-Subjects effect of Type.

APPENDIX C

Table C-1

2(Type: Repeated, Unrepeated) × 6[Timepoint: ln(1-6)] × 2(Math Anxiety: HMA, LMA)

	Dfn	Dfd	F	Cohen's D	p
Between Subjects Effects					
Math Anxiety	1	80	10.65	0.34	0.002
Within Subjects Effects					
Type	1	80	887.31	0.96	4.66E-45
Timepoint	5	400	324.01	0.9	3.38E-138
Type x MA	1	80	2.14	0.16	0.148
Timepoint x MA	5	400	0.62	0.09	0.687
Type x Timepoint	5	400	117.13	0.77	4.60E-76
Type x Timepoint x MA	5	400	5.33	0.25	9.43E-05
Within Subjects Contrasts					
Timepoint	1	80	704.78	0.95	2.02E-41
Timepoint x MA	1	80	0.74	0.1	0.391
Type x Timepoint	1	80	268.98	0.88	2.60E-27
Type x Timepoint x MA	1	80	11.92	0.36	8.91E-04

Table C-1 Notes. Table C-1 shows the results of the primary model from Research Question 3. Note that contrast effects are the natural log of Timepoint [ln(1-6)]. As Type has only two levels, the contrast effect of Type is identical to the Within-Subjects effect of Type.

Table C-2

Repeated Problems: 6[Timepoint: ln(1-6)] × 2(Math Anxiety: HMA, LMA)

	Dfn	Dfd	F	Cohen's D	p
Between Subjects Effects					
Math Anxiety	1	80	5.98	0.26	0.017
Within Subjects Effects					
Timepoint	5	400	372.67	0.91	4.65E-148
Timepoint x MA	5	400	2.98	0.19	0.012
Within Subjects Contrasts					
Timepoint	1	80	664.28	0.94	1.69E-40
Timepoint x MA	1	80	5.36	0.25	0.023

Table C-2 Notes. Table C-2 shows the results of the model for Repeated Problems from Research Question 3. Note that contrast effects are the natural log of Timepoint [ln(1-6)].

Table C-3

Unrepeated Problems: 6[Timepoint: ln(1-6)] × 2(Math Anxiety: HMA, LMA)

	Dfn	Dfd	F	Cohen's D	p
Between Subjects Effects					
Math Anxiety	1	80	11.86	0.36	9.18E-04
Within Subjects Effects					
Timepoint	5	400	93.28	0.73	6.31E-65
Timepoint x MA	5	400	1.86	0.15	0.1
Within Subjects Contrasts					
Timepoint	1	80	222.79	0.86	7.79E-25
Timepoint x MA	1	80	3.67	0.21	0.059

Table C-3 Notes. Table C-3 shows the results of the model for Unrepeated Problems from Research Question 3. Note that contrast effects are the natural log of Timepoint [ln(1-6)].

APPENDIX D

Table D-1

2(Type: Repeated, Unrepeated) × 2(Day: 1, 2) × 2(Math Anxiety: HMA, LMA) × 3[Timepoint: ln(1-3)]

	Dfn	Dfd	F	Cohen's D	p
Between Subjects Effect					
Math Anxiety	1	80	10.65	0.34	0.002
Within Subjects Effects					
Type	1	80	887.31	0.96	4.66E-45
Day	1	80	410.00	0.91	3.18E-33
Timepoint	2	160	301.09	0.89	5.82E-55
Type x MA	1	80	2.14	0.16	0.148
Day x MA	1	80	0.10	0.03	0.757
Timepoint x MA	2	160	1.22	0.12	0.298
Type x Day	1	80	138.59	0.8	3.83E-19
Type x Timepoint	2	160	160.40	0.82	5.93E-39
Day x Timepoint	2	160	185.58	0.84	2.05E-42
Type × MA × Day	1	80	10.66	0.34	0.002
Type × MA × Timepoint	2	160	3.42	0.2	0.035
Day x MA x Timepoint	2	160	0.60	0.09	0.551
Type x Day x Timepoint	2	160	41.31	0.58	3.44E-15
Type x Day x Timepoint x MA	2	160	0.69	0.09	0.501
Within Subjects Contrasts	**Dfn**	**Dfd**	**F**	**Cohen's D**	**p**
Day	1	80	410.00	0.91	3.18E-33
Timepoint	1	80	321.74	0.89	9.15E-30
Day x MA	1	80	410.00	0.91	3.18E-33
Timepoint x MA	1	80	1.34	0.13	0.251
Type x Day	1	80	138.59	0.8	3.83E-19
Type x Timepoint	1	80	172.19	0.83	1.21E-21
Day x Timepoint	1	80	270.69	0.88	2.14E-27
Type × MA × Day	1	80	10.66	0.34	0.002
Type × MA × Timepoint	1	80	2.30	0.17	0.133
Day x MA x Timepoint	1	80	0.86	0.1	0.357
Type x Day x Timepoint	1	80	56.29	0.64	7.57E-11
Type x Day x Timepoint x MA	1	80	0.72	0.09	0.4

Table D-1 Notes. Table D-1 shows the results of the model for Research Question 4. Note that contrast effects are the natural log of Timepoint [ln(1-3)]. As Day and Type have only two levels, the contrast effects of Day and Type are identical to the Within-Subjects effects of Day and Type, respectively.