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COMMENTARY PAPER

Cognitive neuroscience and mathematics learning: how far have we come? Where do we need to go?

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Abstract In this commentary on the ZDM special issue: 'Cognitive neuroscience and mathematics learning—revisited after 5 years', we explore the progress that has been made since ZDM published a similar special issue in 2010. We consider the extent to which future frontiers and methodological concerns raised in the commentary on the 2010 issue by Grabner and Ansari have been addressed 5 years on. We identify areas of progress as well as issues that continue to require additional research and methodological innovation to make further progress. Finally, we discuss future directions that could lead to significant progress in the interdisciplinary crossroads between cognitive neuroscience and mathematics learning over the next 5 years.

Keywords Mathematics education · Neuroscience · Cognition · Educational neuroscience

1 Introduction

In 2010, ZDM published a special issue entitled 'Cognitive Neuroscience and Mathematics Learning'. This special issue comprised a set of empirical research papers that addressed key questions in the cognitive neuroscience of mathematics learning, using multidisciplinary, innovative approaches. Since 2010, the number of studies addressing problems in mathematics education using cognitive neuroscience methods has grown substantially. In view of this growth in research, ZDM is now publishing another special issue on this important topic entitled: "Cognitive neuroscience and mathematics learning—revisited after 5 years," edited by Roland Grabner and Bert De Smedt.

The aim of this commentary is to reflect on what progress has been made in the 'Cognitive Neuroscience of Mathematics Education' since the publication of a special issue in ZDM in 2010 and to address the extent to which the issues raised in the 2010 commentary by Grabner and Ansari (that reflected on the work presented in the original ZDM special issue) have been addressed.

A review of the articles that comprise the present special issue reveals that the field has grown substantially and that cutting-edge questions are being addressed using a diverse set of methods, including eye-tracking, EEG and functional neuroimaging. It is clear that there is considerable energy behind efforts attempting to connect research in cognitive neuroscience with issues in mathematics education, both in terms of specific research projects and broader knowledge exchange between the two fields.

In their 2010 commentary on the 'Cognitive Neuroscience of Mathematics Education' special issue, Grabner and Ansari identified two key issues:

1. Selection of samples: Grabner and Ansari noted that the majority of the studies in the 2010 ZDM special issue had adults as their study participants. They argued that studying adults can only take us some way toward a cognitive neuroscience of mathematics learning. In particular, there is a need to study samples of children at the ages/grades when they are acquiring particular mathematical skills.
2. Ecological validity: Grabner and Ansari noted that there is a need to increase the ecological validity of the testing situations and specific tests used to meas-

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ure mathematical processing. Specifically, they argued that many of the papers contained within the 2010 special issue of ZDM used highly controlled experimental procedures in the tradition of Experimental Psychology. While such studies are desirable from a methodological point of view, they may not resemble what is going on inside students' heads when they are sitting in a mathematics classroom. Greater ecological validity may mean using protocols that are less tightly controlled, but will connect more closely with the contexts within which mathematical learning and thinking occurs.

2 How much progress has been made since 2010?

With respect to the issues raised by Grabner and Ansari in 2010, much progress has been made in the diversity of topics being investigated that make connections between cognitive neuroscience and mathematics education. The current special issue contains articles on an impressive diversity of topics—including fraction comparison, geometry, arithmetic and artificial symbol learning, to name just a few. In comparison to the state of the art in 2010, a more diverse set of questions pertaining to mathematics education is being investigated from a cognitive neuroscience perspective. This represents significant and exciting progress. We also note that there is growing diversity in terms of the academic backgrounds and countries from which the authors of the papers in the present special issue come from, showing that efforts to bridge cognitive neuroscience and education are not only becoming increasingly multidisciplinary, but also that this is now a global research agenda.

However, the concerns raised by Grabner and Ansari are still relevant in this emerging field of inquiry. In particular, the majority of papers published in the special issue (6 of 9) present experiments that had adults as the research participants. This is problematic for the same reasons that were highlighted by Grabner and Ansari (2010). Moreover, there is still a concern over ecological validity. Many of the experiments described in the articles contained within this current special issue, published over 5 years after the 2010 issue, are well-controlled psychological experiments, but their connections to the educational context and the mathematics classroom are unclear. For example, Schillinger et al. (2016) use a classic numerical Stroop task, Vogel et al. (2016) use a standard number-line judgment task, and Merkley et al. (2016) and Pollack et al. (2016) use artificial symbol-learning paradigms. Each of these paradigms has a long and well-established history within the field of psychology and in the neuroimaging literature. This makes their findings more directly interpretable from a cognitive neuroscience perspective. However, it is harder to see how these stock cognitive tasks directly relate to classroom activities. That said, some of the studies do go some way towards greater ecological validity. For instance, Schillinger attempted to simulate high-stress testing situations by using a pressure-inducing manipulation. Vogel et al. modified the number-line judgment task to be more applicable to health-related assessments. Other studies in this issue went still further, by basing the stimuli for their experimental paradigms directly on material that is typically seen in actual classrooms and exams (e.g., Babai et al. 2016; Leikin et al. 2016; Waisman et al. 2016).

In addition to the issues raised by Grabner and Ansari (2010), there are a number of other points worthy of discussion and consideration when thinking about how best to pursue future research that connects cognitive neuroscience and mathematics education. Specifically, the articles in the current special issue reflect a substantial difficulty in bridging levels of analysis. First of all, when neuroimaging is used, it is often very difficult to understand the connection between the behavioral and brain-imaging data. Indeed, of the six papers that report neural data in this issue, all six report behavioral or neural data in isolation of one another; and while connections are sometimes drawn in the conclusion sections, these connections are not directly investigated. A few papers even report divergent (e.g., Merkley et al. 2016) or even somewhat contradictory (e.g., Leikin et al. 2016) behavioral and neural patterns of results. It is also clear from these papers that there are behavioral findings that have clear implications for education.

For example, Merkley et al.'s results highlight the potential importance of ordinality for learning novel numerical symbols, and Babai et al. (2016) show that how children acquire an understanding of the concept of perimeter depends critically on the order in which different types of examples (using continuous vs. discrete components) are presented. However, in the case of Merkley et al., it is unclear precisely how the ERP results might prove useful to an educator, and Babai et al.'s study was purely behavioral and therefore it is unclear what neuroscience would add to these findings. In other words, interesting as the behavioral results of these studies are, they also highlight the difficulty, as noted above, in understanding exactly how and what neuroimaging data can tell us about education. These findings are reflective of a disconnect that exists throughout the literature and illustrate how difficult it can be to connect levels of analysis and to draw conclusions about education from purely neuroscientific data. These problems of connecting across levels of explanation are common in cognitive neuroscience. One would expect that more research, like the evidence reported in this special issue, will help further bridge levels of explanation. That said, there is a great need, as the field continues to progress over the next 5 years, to pay more attention to the ways in which we might integrate across levels of explanation in order to truly benefit from both neuronal and behavioral data in our empirical study of issues which are specifically relevant to mathematics education.

Even if one leaves the issue of connecting across multiple levels of explanation aside, there are also great difficulties in interpreting the meaning and significance of neuroimaging findings. A prime example is how to interpret greater or lesser 'activation' (e.g., in terms of % change in the fMRI BOLD signal, microvolt differences in a given ERP component, or differences in the amount of power in various EEG frequency bands). A critical challenge is to translate these differences in neural signal into not just behavioral terms but also into educationally meaningful concepts. For example, Waisman et al. (2016) found that gifted students showed reduced amplitude ERP modulation when solving geometry problems. While an interesting theoretical result, the authors interpreted this result in terms of neural 'efficiency'—gifted students solved the problems in a neurally more efficient manner. Though this sort of interpretation is common throughout the cognitive neuroscience literature, the term 'efficiency' may be of limited use. Poldrack (2015) discusses how this term adds very little explanatory power on its own from a theoretical standpoint (precisely how reduced activation corresponds to greater output per energy unit expended often remains unspecified). Similarly, from an educational standpoint, does this mean that a given child is working less hard, so to speak, to achieve equal or better academic outcomes? How are they doing so—by using more efficient strategies, by accessing a more extensive existing knowledge set, etc.? Our point here is not to level criticism directly at Waisman and colleagues; as Poldrack (2015) points out, this is a wide-spread issue throughout the field of (especially developmental) cognitive neuroscience (and indeed, within this issue, Leikin et al. 2016, interpret key aspects of their results in terms of neural efficiency as well). Instead, our point is that perhaps more would be gained by linking these changes in neural activity to specific behaviors (such as strategy use, a priori expertise levels, and so on) that may be more directly translated into points of contact that carry import to educators—such as relative emphasis in curriculum design, creation and interpretation of screening tools, and so forth.

In a similar vein, Spüler et al. (2016) report an intriguing set of results in which different levels of arithmetic task difficulty could be classified based on key features of EEG time–frequency data. Here again, though, it is perhaps difficult to see precisely how this is of direct benefit to educators. One essential question is precisely what is meant by 'difficulty'. Many factors are known to modulate performance (i.e., in terms of higher error-rates and longer response-times) on arithmetic problems, such as numerical size, familiarity —and hence retrievability— of specific problems, cues to certain 'short-cut' strategies, presence or absence of carry/borrow operations, and so on (for a review, see Ashcraft 1995; see also, LeFevre et al. 1996; Logie et al. 1994). Perhaps by identifying which of these *problem* features were contributing to classification of 'difficulty' based on neural features, the authors might have been better placed to translate their work into something more directly amenable to understanding how best to teach arithmetic.

While the special issue contains many studies that address important questions regarding the neural correlates of the basic cognitive principles that underlie mathematical processing in the brain, few of the papers address questions that come directly from mathematics education. Understandably, the connection between cognitive neuroscience and mathematics education is currently driven by traditional experimental psychology paradigms, such as the numerical Stroop task (Schillinger et al. 2016), number line judgments (Vogel et al. 2016), fraction-magnitude comparisons (Obersteiner and Tumpek 2016), and artificial learning paradigms (Merkley et al. 2016; Pollack et al. 2016). This can lead to a heavy emphasis on very basic representations and processes that, while no doubt critical for understanding mathematics processing from a theoretical perspective, may nevertheless fail to directly address critical questions relevant to mathematics educators. In order to advance the field and increase the relevance of the issues addressed for mathematics education, more empirical questions need to be derived from current issues in education, such as the debate over whether mathematics learning benefits from teaching procedures vs. concepts, whether one approach to teaching fractions is more beneficial than another, or whether teaching using spatial strategies is more efficacious than teaching using verbal strategies, etc. In sum, more research needs to focus on questions that come from within education.

3 Future directions

Looking ahead, there are topics/methodological approaches that are absent (or only addressed briefly) in the collection of papers that comprise this new special issue, which would, in our view significantly advance the field. Here we highlight two that we feel are particularly important future frontiers of the study of mathematics education using a cognitive neuroscience approach.

4 Studying the effects of educational interventions using neuroimaging data

Combining the study of educational interventions with functional and structural neuroimaging allows researchers to understand how brain function and even structure changes as children strengthen existing or acquire new skills and knowledge. In the domain of reading and the study of developmental dyslexia, there now exists a large body of research combining neuroimaging with intervention research (e.g., Eden et al. 2004; Shaywitz et al. 2004). To the best of our knowledge, there is only one study that combined research on intervention with neuroimaging to help children with Developmental Dyscalculia (Kucian et al. 2011).

This approach allows for researchers to get closer to understanding the neurobiological changes caused by learning and thereby allows researchers to directly connect changes in brain with changes in behavior. Furthermore, understanding which brain circuits are changed by an educational intervention can help us to better understand the mechanisms that underpin the change in the student's behavior. For example, there is currently both interest in and controversy around whether interventions focusing on more visual, imagery-based or verbal, retrieval-based strategies are more effective in mathematics education. However, knowing whether students actually adopt these strategies is often restricted to subjective self-reports that can easily be biased. Neuroimaging can provide a more objective assessment of how students' strategies may be changing. For instance, an imagery-based strategy might predict an increase in the distinctiveness in neural patterns for different mathematics problems with imagery training in areas related to visual processing. A retrieval-based strategy might predict an increase in the distinctiveness in neural patterns for different mathematics problems with retrieval training in areas related to verbal processing. Indeed, even the match or mismatch between subjective reports and objective brain data—and how this mis/match relates to actual mathematics learning might provide critical insight into how children actually learn new mathematics concepts. This in turn could have important implications for the ongoing debate about how best to teach these concepts. In this way neuroimaging can be an additional, valuable tool in the assessment and continual refinement of evidence-based, educational interventions.

Furthermore, using neuroimaging to study the effects of educational interventions on the brain can also be used to contrast different pedagogical approaches to teaching the same skills and/or concepts. By using neuroimaging (in addition to behavioral measures) to contrast different types of interventions, commonalities as well as differences in their underlying mechanisms that lead to changes in students' ability and understanding can be better understood.

5 Studying the utility of neuroscience methods to predict individual differences

Screening children early is crucial to identifying those who might be at risk of developing learning difficulties. This may help to pave the way toward providing interventions designed to prevent/reduce deleterious long-term developmental trajectories caused by such difficulties. Recent evidence, particularly from the study of developmental difficulties in acquiring literacy skills (i.e., Developmental Dyslexia) have suggested that neuroimaging measures are not only responsive to intervention but can also be sensitive tools for the prognosis of long-term developmental outcomes (e.g., Hoeft et al. 2007, 2011). Importantly, in some cases, neuroimaging measures explain more variability in future outcomes than concurrently acquired behavioral measures. This suggests that neuroimaging methods can represent a significant 'added value' when it comes to screening children who might be at risk of developing long term learning difficulties. In the domain of mathematics, there also exists an emerging body of research demonstrating the prognostic utility of neuroimaging for long-term individual differences in mathematical abilities as well as in responses to intervention (Supekar et al. 2013; Evans et al. 2015). Future studies should further examine the value offered by neuroimaging as a screening measure. Here, it would be particularly useful to examine the use of lower-cost neuroimaging methods such as EEG and Near Infrared Spectroscopy (NIRS) as predictors of long-term outcomes. Such studies could, in the long term, lead to real innovations in early screening for later difficulties.

6 Conclusions

In the ensuing 5 years since the publication of the ZDM special issue on 'Cognitive Neuroscience and Mathematics Learning,' the field has matured. The papers contained within the present special issue reveal a greater diversity of research topics as well as methodological approaches than was the case in 2010.

While there has been undoubted progress, many of the issues that were highlighted in the commentary by Grabner and Ansari in the 2010 ZDM special issue continue to be major challenges that need to be tackled as we take the field forward. There is a need to move beyond the study of adults to studying problems in mathematics education in populations of young learners. Such research will increase the connection between cognitive neuroscience and mathematics education. Similarly there is a need to move beyond the use of traditional paradigms that are derived from experimental psychology towards more ecologically valid research paradigms. Moreover, the contributions in the present special issue reveal that much work remains to be done in connecting the behavioral and neural levels of explanation, and to develop models in which they can truly combine to yield a better understanding of issues specifically relevant to mathematics education.

In 2010, Grabner and Ansari suggested that one key to making progress in this field is to encourage greater collaboration between mathematics education researchers and cognitive neuroscientists. The need for more such collaboration persists. This not only requires the willingness of researchers from both backgrounds to collaborate, but also highlights the need for funding agencies and institutions to encourage such collaboration by offering infrastructure, funding and training programs that support them.

Interdisciplinary research, while extremely rewarding, is also extremely difficult. Such research further increases in complexity when demands are made not only to create new knowledge but also to translate such knowledge into viable applications. As the saying goes, 'Rome wasn't built in a day'. The present special issue represents an important milestone in the long-term quest to build bridges between cognitive neuroscience and mathematics education research and practice. Undoubtedly, fueled by the progress reflected in this special issue, there will be great progress over the next 5 years.

References

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: a review and summary of new directions. *Mathematical Cognition, 1*(1), 3–34.

Babai, R., Nattiv, L., Stavy, R. (2016). Comparison of perimeters: improving students' performance by increasing the salience of the relevant variable. *ZDM Mathematics Education*, 48(3), this issue.

Eden, G. F., Jones, K. M., Cappell, K., Gareau, L., Wood, F. B., Zeffiro, T. A., & Flowers, D. L. (2004). Neural changes following remediation in adult developmental dyslexia. *Neuron, 44*(3), 411–422. doi:10.1016/j.neuron.2004.10.019.

Evans, T. M., Kochalka, J., Ngoon, T. J., Wu, S. S., Qin, S., Battista, C., & Menon, V. (2015). Brain structural integrity and intrinsic functional connectivity forecast 6 year longitudinal growth in children's numerical abilities. *Journal of Neuroscience, 35*, 11743–11750.

Grabner R. H., & Ansari D. (2010). Promises and potential pitfalls of a 'cognitive neuroscience of mathematics learning'. *ZDM: the international journal on mathematics education, 42*(6), 655–660.

Hoeft, Fumiko, McCandliss, B. D., Black, J. M., Gantman, A., Zakerani, N., Hulme, C., & Gabrieli, J. D. E. (2011). Neural systems predicting long-term outcome in dyslexia. *Proceedings of the National Academy of Sciences of the United States of America, 108*, 361–366. doi:10.1073/pnas.1008950108.

Hoeft, F., Ueno, T., Reiss, A. L., Meyler, A., Whitfield-Gabrieli, S., Glover, G. H., & Gabrieli, J. D. (2007). Prediction of children's reading skills using behavioral, functional, and structural neuroimaging measures. *Behavioral Neuroscience, 121*(3), 602–613.

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., et al. (2011). Mental number line training in children with developmental dyscalculia. *NeuroImage, 57*(3), 782–795.

LeFevre, J. A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: reassessing the problem size effect in adults. *J Exp Psychol Learn Mem Cognit, 22*(1), 216–230.

Leikin, R., Waisman, I., Leikin, M. (2016). Does solving insight-based problems differ from solving learning-based problems: some evidence from an ERP study. *ZDM Mathematics Education*, 48(3), this issue.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Mem Cognit, 22*(4), 395–410.

Merkley R, Shimi A, Scerif G (2016). Electrophysiological markers of newly acquired symbolic numerical representations: the role of magnitude and ordinal information. *ZDM Mathematics Education*, 48(3), this issue.

Obersteiner A, Tumpek C (2016). Measuring fraction comparison strategies with eye-tracking. *ZDM Mathematics Education*, 48(3), this issue.

Poldrack, R. A. (2015). Is "efficiency" a useful concept in cognitive neuroscience? *Developmental Cognitive Neuroscience, 11*, 12–17.

Pollack C, Geurrero SL, Star JR (2016). Exploring mental representations for literal symbols using priming and comparison distance effects. *ZDM Mathematics Education*, 48(3), this issue.

Schillinger FL, De Smedt B, Grabner RH (2016). When errors count: an EEG study on numerical error monitoring under performance pressure. *ZDM Mathematics Education*, 48(3), this issue.

Shaywitz, B. A., Shaywitz, S. E., Blachman, B. A., Pugh, K. R., Fullbright, R. K., Skudlarski, P., & Gore, J. C. (2004). Development of left occipitotemporal systems for skilled reading in children after a phonologically- based intervention. *Biological Psychiatry, 55*(9), 926–933. doi:10.1016/j.biopsych.2003.12.019.

Spüler, M., Walter, C., Rosenstiel, W., Gerjets, P., Moeller, K., Klein, E. (2016). EEG-based prediction of cognitive workload induced by arithmetic: a step towards online adaptation in numerical learning. *ZDM Mathematics Education*, 48(3), this issue.

Supekar, K., Swigart, A. G., Tenison, C., Jolles, D. D., Rosenberg-Lee, M., Fuchs, L., & Menon, V. (2013). Neural predictors of individual differences in response to math tutoring in primary-grade school children. *Proceedings of the National Academy of Sciences of the United States of America, 110*(20), 8230–8235. doi:10.1073/pnas.1222154110.

Vogel, S. E., Keller, C., Koschutnig, K., Reishofer, G., Ebner, F., Dohle, S., Siegrist, M., Grabner, R. H. (2016). The neural correlates of health risk perception in individuals with low and high numeracy. *ZDM Mathematics Education*, 48(3), this issue.

Waisman, I., Leikin, M., Leikin, R. (2016). Brain activity associated with logical inferences in geometry: focusing on students with different levels of ability. *ZDM Mathematics Education*, 48(3), this issue.