CHAPTER

# On the ordinality of numbers: A review of neural and behavioral studies

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## Abstract

The last several years have seen steady growth in research on the cognitive and neuronal mechanisms underlying how numbers are represented as part of ordered sequences. In the present review, we synthesize what is currently known about numerical ordinality from behavioral and neuroimaging research, point out major gaps in our current knowledge, and propose several hypotheses that may bear further investigation. Evidence suggests that how we process ordinality differs from how we process cardinality, but that this difference depends strongly on context—in particular, whether numbers are presented symbolically or nonsymbolically. Results also reveal many commonalities between numerical and nonnumerical ordinal processing; however, the degree to which numerical ordinality can be reduced to domain-general mechanisms remains unclear. One proposal is that numerical ordinality relies upon more general short-term memory mechanisms as well as more numerically specific long-term memory representations. It is also evident that numerical ordinality is highly multifaceted, with symbolic representations in particular allowing for a wide range of different types of ordinal relations, the complexity of which appears to increase over development. We examine the proposal that these relations may form the basis of a richer set of associations that may prove crucial to the emergence of more complex math abilities and concepts. In sum, ordinality appears to be an important and relatively understudied facet of numerical cognition that presents substantial opportunities for new and ground-breaking research.

## Keywords

Numerical ordinality, Nonnumerical ordinality, Intraparietal sulcus (IPS), Neuroimaging, Numerical cognition, Ordinal number development

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## 1 GENERAL INTRODUCTION

The last few decades have seen remarkable growth in efforts to better understand how the human mind and brain process numbers and numerical information. When one mentions "numbers," most literate individuals tend to think of number symbols–number words ("one," "two," "three," …) or their corresponding written symbols (often, Indo-Arabic numerals, 1, 2, 3, …). There is also substantial evidence that humans and many other species can represent nonverbal quantities or magnitudes. For instance, without explicitly counting, one can estimate approximately which of two bushes contains more berries, or which of two tribes contains more members (Dehaene, 1997). This amount or quantity is a property of a set of objects and is typically referred to as *cardinality*. Cardinality is the answer to the question, "How many?" One can answer the question approximately, as in the earlier examples, or, perhaps, as is more familiar to most readers, cardinality can be assessed by counting (the cardinality of a set is the last number one says when counting up the comprising members of said set). A second important property of numbers is *ordinality*; ordinality answers the question, "What position (or rank)?" Ordinality is a property of individual members of a set (the *first* runner, the *second* runner, etc.) in relation to the other members of that set. Interestingly (and perhaps crucially), cardinal values can also be ordered—for instance, one can order numbers in terms of their cardinality (eg, in the set {1, 5, 12}, 1 is 1st, 5 is 2nd, 12 is 3rd; this illustrates that ordinal position and cardinal value need not be perfectly aligned; see also Fig. 1). It is in this way that ordinality and cardinality are often intertwined, but as we shall see, they are also dissociable, not only conceptually, as in Fig. 1, but also in terms of how humans actually process ordinal and cardinal aspects of numbers.

The focus of this review is the ordinality of numbers. The substantial majority of previous neural and behavioral work on how we process numbers has focused primarily on the cardinality of numbers. Recent years, however, have seen a steady uptick in work focusing on numerical ordinality. This work has begun to shed new light not only on how we process ordinality in basic numerical contexts, but is also beginning to reveal that ordinality is crucial for understanding how we process more abstract arithmetical and mathematical relations that make math such a powerful tool. In the present review, we examine this recent upsurge in work on numerical ordinality. A central conclusion of this review is that, simple as it may seem, ordinal processing of numbers is in fact both complex and multifaceted.

We begin by providing an overview of research examining similarities and differences between ordinal and cardinal processing. We conclude that the two are distinct, though the extent of this distinction likely depends on several contextual factors. One such factor is whether the quantities being processed are represented symbolically (eg, numerals) or nonsymbolically (eg, dot arrays). We next summarize findings from several domains of ordinal processing that, while not necessarily numerical (eg, letters of the alphabet or days of the week), may nevertheless prove useful or even crucial for understanding the representation of order in numerical

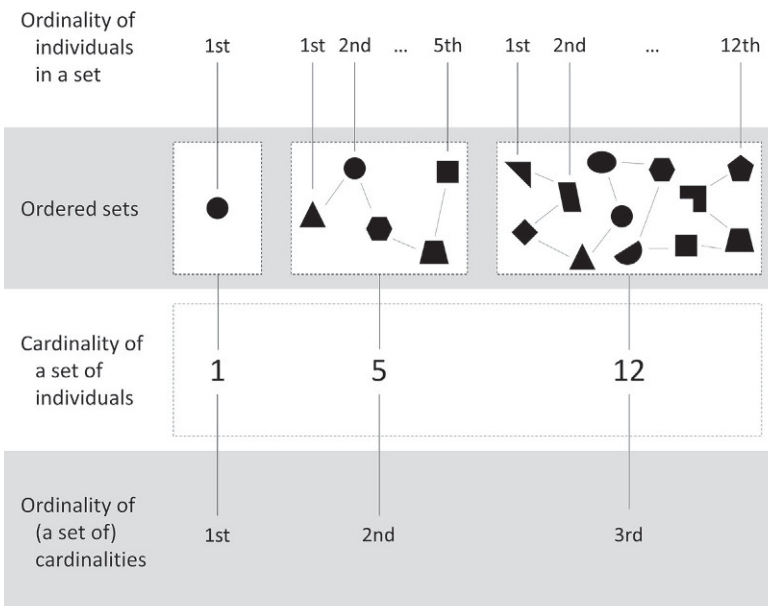


**FIG. 1**

Schematic illustration of ordinality and cardinality.

sequences. While we see clear relationships between numerical and nonnumerical ordinal processing, evidence is either mixed or incomplete with respect to the question of whether numerical ordinal processing is "merely" reducible to domain- or stimulus-general processing of ordinal sequences. We also examine work that has begun to unpack the underlying cognitive and neural mechanisms that support the processing of numerical order, highlighting this as one potential area with significant opportunity for ground-breaking research. In the final section, we turn to how ordinality may prove to be a crucial piece in understanding the acquisition of the symbolic representation of numbers, as well as acquisition of more sophisticated forms of numerical processing, such as mental arithmetic. In particular, we propose that numerical order may play a key role in allowing symbolic representations of numbers to go beyond the cardinal values they represent. Moreover, the multifaceted nature of ordinal processing may in turn be a key mechanism by which we go beyond the item–item associations linking the count-list to the richer network of associations that comprise a more sophisticated system of numerical thought. Throughout the review, we scrutinize the limitations of the current work on numerical ordinality and suggest future avenues of research that might test and address the gaps and weaker points in our current understanding of how numerical ordinality is processed, and what a better understanding might mean for numerical and mathematical cognition.

## 2 HOW DIFFERENT ARE ORDINALITY AND CARDINALITY?

Numbers convey different meaning in different contexts. As noted in the introduction (see also Fig. 1), the Indo-Arabic numeral 3, depending on the context, may refer to three apples (cardinality), or to the third runner in a marathon (ordinality). As adults, we are able to shift between these different numerical contexts, suggesting a differentiation in the way we represent ordinal and cardinal numbers. In this section, we review empirical evidence that yields insight into the similarities and differences between how we actually process ordinality and cardinality. One important conclusion is that ordinal and cardinal processing may be particularly distinct for symbolic representations of number.

### 2.1 ORDINAL AND CARDINAL PROCESSING IN THE BRAIN

Neuropsychological case studies have demonstrated selective impairments of ordinality and cardinality in brain-damaged patients (Delazer and Butterworth, 1997; Turconi and Seron, 2002). Delazer and Butterworth (1997) reported the case of a 56-year-old patient SE who suffered from a left frontal infarct. SE showed severe impairments in arithmetic and number comparison ("Which of two presented numerals is numerically larger?"—ie, relative cardinality). However, SE demonstrated no difficulties in producing number sequences (either with spoken numbers words or written Arabic numerals), counting dots, or naming or writing the correct number when asked "Which number comes next?" In other words, SE showed deficits in cardinal aspects of number processing, but relatively preserved ordinal processing of numbers. The opposite pattern was observed in patient CO who suffered from lesions in the left posterior parietal cortex and the right parietal occipital junction (Turconi and Seron, 2002). CO showed little impairment in comparing numbers or in judging the correct position of numerals on an analogue scale. However, CO demonstrated severe difficulties when judging ordinal relationships between numbers, letters of the alphabet, days of the week, and months of the year. He was unable to indicate whether a number comes before or after 5, whether a letter comes before or after the letter M, or whether a day comes before or after Wednesday. The selective impairment of either the ordinal or the cardinal meaning of numbers (ie, double-dissociation) in brain-damaged patients, provides strong evidence that the processing of ordinality and cardinality are associated with different computational systems within the human brain. However, this observed dissociation in brain-damaged patients does not necessarily imply that ordinal and cardinal representations are functionally unrelated in intact brains.

Using functional magnetic resonance imaging (fMRI) with healthy adult participants, Lyons and Beilock (2013) examined ordinal and cardinal processing of symbolic numbers (Indo-Arabic numerals), nonsymbolic numbers (dot arrays), and nonnumerical magnitudes (luminance). For ordinal tasks, participants determined whether three stimuli were in left–right order (increasing or decreasing—eg, 1-2-3, 3-2-1) vs not in order (eg, 1-3-2, 3-1-2). For cardinal tasks, participants determined which of two stimuli was numerically greater (or brighter in the luminance control tasks). *Numerical* processing was isolated by subtracting activity associated with the relevant luminance task (ordinal or cardinal; visual features—numerals or dots—were also matched) from the numerical task. Results showed a highly similar frontoparietal network was active for ordinal and cardinal judgments specifically for *nonsymbolic* quantities (dot arrays; Fig. 2A). By contrast, there was no overlap anywhere in the brain (even at highly liberal thresholds) between ordinal and cardinal judgments for *symbolic* numbers (ie, numerals). This suggests that, while ordinal and cardinal processing may be closely linked for nonsymbolic quantities such as arrays of dots, this is less the case for symbolic numbers. In general, it appears, from both the neuropsychological work with brain-damaged patients as well as more recent functional neuroimaging work with healthy participants that processing the cardinality and ordinality of numerical symbols relies on different brain circuits.

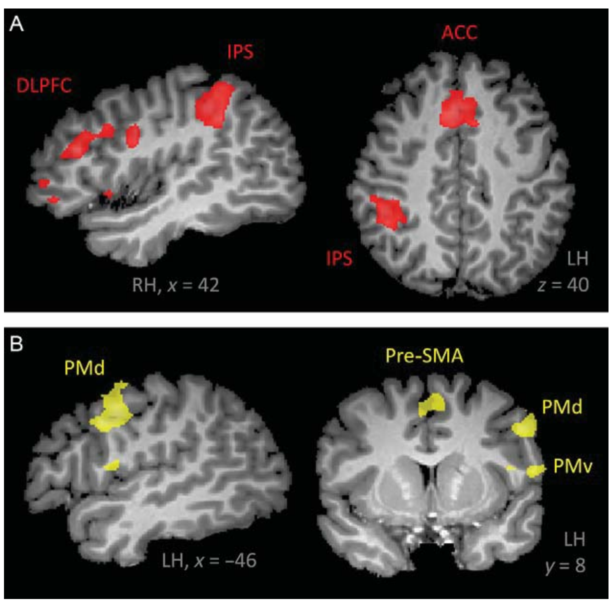


**FIG. 2**

(A) A common, right-lateralized frontoparietal network for nonsymbolic ordinal and cardinal processing. Regions are the conjunction of dot-ordering greater than luminance-ordering and dot-comparison greater than luminance-comparison. Note that no regions showed a similar conjunction of symbolic ordinal and cardinal processing. (B) Left premotor regions specifically activated for the symbolic number ordering.

*Adapted from Lyons, I.M., Beilock, S.L., 2013. Ordinality and the nature of symbolic numbers. J. Neurosci. 33 (43), 17052–17061.*

## 2.2 DISTANCE EFFECTS: DIFFERENT SIGNATURES OF ORDINAL AND CARDINAL PROCESSING

Both behavioral and neuroimaging work have converged to indicate that a key signature of number processing—the distance effect[a]—also distinguishes cardinal and ordinal number processing. However, this distinction is not perfect: it may depend, as we saw in the previous section, on symbolic vs nonsymbolic presentation format, as well as the exact paradigm used. Thus, we see that ordinal and cardinal processing of numbers are intertwined to some extent, but important distinctions can and should be made in pursuit of a complete understanding of how we process numbers.

While canonical distance effects are typically seen for cardinal judgments (eg, Buckley and Gillman, 1974; Moyer and Landauer, 1967), distance effects are sometimes reversed for ordinal judgments: one is faster to verify that (4 ■ 5) is in order than (3 ■ 6) (Turconi et al., 2006). The reversal of the distance effect has since been replicated across different studies (Franklin and Jonides, 2009; Franklin et al., 2009; Goffin and Ansari, 2016; Lyons and Beilock, 2013) and is particularly robust when participants are asked to judge the order of presented number triplets (eg, 1-2-3) instead of number pairs. Reversed distance effects thus provide a clear behavioral signature that is qualitatively distinct for ordinal and cardinal processing, which is consistent with the notion that the two forms of numerical processing differ in important ways, as discussed in the previous section. Specifically, it has been argued that reverse distance effects indicate order-specific mental operations, which are distinct from cardinality discrimination, and may be reflective of a mechanism that enables a fast recognition of successively ordered numbers (Franklin et al., 2009; Lyons and Beilock, 2013; Turconi et al., 2006). Consistent with this notion, Goffin and Ansari (2016) demonstrated that canonical and reverse distance effects are uncorrelated across subjects.

Neuroimaging evidence is also consistent with the notion that reversed vs canonical distance effects indicate a qualitative distinction between ordinal and cardinal numerical processing, respectively. Using only symbolic stimuli, Franklin and Jonides (2009) had participants either judge the relative order of number triplets (ordinal processing) or the relative magnitude of number pairs (cardinal processing). Consistent with the work reviewed above, they found reverse distance effects for correctly ordered triplets in the ordinal task and canonical distance effects in the cardinal task. Substantial prior work has implicated the intraparietal sulci (IPS) in numerical processing in general (for a review, see Ansari, 2008; Nieder and Dehaene, 2009); however, prior to Franklin and Jonides, primarily cardinal judgments had been assessed using fMRI. The authors found overlapping distance effects for ordinal

[a]The classic, or canonical, distance effect is obtained when one asks participants (adults, children, nonhuman animals, etc.) to determine the relative cardinality of two numbers (symbolic or nonsymbolic). Results consistently show that participants perform worse—slower response times and higher error rates—when the absolute numerical distance between the two numbers is smaller (e.g., 4 ■ 5 is harder than 3 ■ 6).

and cardinal judgments in the IPS, consistent with the notion that this region plays a key role in numerical processing more generally. Crucially, however, this overlap was only seen when the direction of the contrast for the ordinal task was reversed: for the cardinal task, greater IPS activity was seen for close relative to far distances; for the ordinal task, greater IPS activity was seen for far relative to close distances. In other words, activity in a brain region typically seen as central to number processing was highly dependent on whether the task context was ordinal or cardinal in nature, consistent with the need to distinguish these two aspects of how we process and understand numbers.

It is important to acknowledge, however, that the reversal of distance effects tends to be highly sensitive to context. For instance, reverse distance effects tend to be seen only when the stimuli are in fact correctly ordered (eg, performance is better when verifying that 3-4-5 is in order relative to 2-4-6). One in fact typically sees canonical distance effects for unordered sets (eg, one is faster to reject 4-6-2 than 4-5-3 as not in order). This suggests that, as noted earlier, ordinal and cardinal processing, while distinct, are not entirely unrelated. However, Vogel et al. (2015) found that—at least in first graders—even canonical distance effects in ordinal judgments were uncorrelated with canonical distance effects in cardinal judgments.

In a similar vein, Turconi et al. (2004) also demonstrated canonical distance effects for both ordinal and cardinal judgments when comparing numbers to a standard held in mind. Participants performed a number comparison task (is the presented number larger or smaller than 15) and a number ordinal judgment task (does the presented number come before or after 15). The authors also recorded event-related potentials (ERPs). ERP analyses demonstrated significant canonical distance effects for both ordinal and cardinal judgments at the P2 component over parietal electrodes; however, significantly shorter latencies and greater amplitudes were found for the cardinal task. Moreover, the ordinal task showed a significant canonical distance effect over right parietal electrodes and a significant amplitude difference over prefrontal regions at the P3 component (cardinal judgments showed neither effect). Thus, in spite of showing canonical distance effects for the ordinal task, both Vogel et al. (2015, discussed earlier) and Turconi et al. (2004) found further evidence that ordinal and cardinal judgments elicit distinct processes.

Given the sensitivity of distance effects to context in ordinal judgments, it is worth noting that Vogel et al. (2015) used pairs instead of triplets for their ordinal task (ie, similar to Turconi et al., 2006). In contrast to Turconi et al., however, Vogel et al. found a canonical distance effect for the ordinal task—even for ordered pairs. One obvious discrepancy between the two studies is that Turconi et al. examined adult participants and Vogel et al. tested first graders. Interestingly, Lyons and Ansari (2015) also examined first graders, but, using a triplet version of the ordering task, they found robust reverse distance effects. This suggests that for certain versions of ordinal judgments, task and participant parameters may play an important role in determining the pattern of results with respect to distance effects. One possibility is that certain task parameters may bias some participants to emphasize primarily ordinal vs cardinal strategies, or the other way around. On the one hand, this highlights the highly contextually dependent nature of number processing; but it also clearly indicates a need for future research to better unpack the relevant parameters and constraints as they pertain to ordinal processing of numbers in particular.

It is also worth noting that there is some discrepancy regarding precisely for which distances the distance effects are reversed in ordinal judgments. Examining adult subjects with number pairs, Turconi et al. (2006) reported reverse distance effects only for distance 1; that is, performance on ordered trials with a distance of 1 (4 ■ 5) was better than on trials with distance 2 (3 ■ 5), but this pattern did not extend to larger distances. Using triplets and also with adult participants, Goffin and Ansari (2016) found a similar result. One possibility is that reverse distance effects are limited only to numbers that are adjacent in the count sequence. However, Franklin and Jonides (2009; see also Franklin et al., 2009, for a similar result) found robust distance effects (using triplets with adult subjects) for stimuli that were not strictly adjacent; eg, (22-23-25) and (25-23-22) were verified as being in order more rapidly than (22-26-28) and (28-26-22).[b] Furthermore, in 1st–6th grade children, Lyons and Ansari (2015) found that the pattern of a reverse distance effect for ordered trials (using triplets) extended out to distances of three for both single- and double-digit trials. Finally, Franklin et al. (2009) found that reverse distance effects were particularly strong when crossing a category boundary (decades for numbers, years for months), a pattern that was also seen in Lyons and Ansari (2015). Hence, on the one hand, it does not seem to be the case that reverse distance effects can be written off as pertaining "just" to adjacent items in the count sequence; on the other hand, the precise circumstances in which it obtains (and the implications this may have for the underlying mechanisms behind ordinal processing) remain unclear. Though we return to this issue in a later section, the need for further work in this area is evident.

## 2.3 SYMBOLIC VS NONSYMBOLIC ORDINAL PROCESSING

Another important context that may modulate distance effects in numerical ordinal judgments is format—specifically, whether numbers are presented symbolically or nonsymbolically. Consistent with several studies reviewed earlier, Lyons and Beilock (2013) showed reverse distance effects for ordinal judgments over ordered sets of number symbols (numerals). In contrast, only canonical distance effects were found for nonsymbolic quantities (dot arrays), regardless of context (ordered or not ordered). This is consistent with the authors' fMRI results indicating that symbolic and nonsymbolic number ordering are distinct. To the best of our knowledge, reverse distance effects have only been found when assessing the ordinality of symbolic stimuli, which suggests the distinction between ordinal and cardinal processing may be especially pronounced for number symbols.

[b]Note that the presence of reverse distance effects for increasing and decreasing trials was also found for triplets in Lyons and Beilock (2013).

This distinction is broadly consistent with an influential review by Marshuetz and Smith (2006), who suggested that both magnitude and associative (or retrieval-based) mechanisms play a role in ordinal processing more generally. One critical factor may be whether stimuli are represented as nonsymbolic, approximate magnitudes or symbols. Specifically, while ordinal processing of approximate, nonsymbolic inputs may depend crucially on a magnitude (ie, cardinality)-based mechanism, the same may not be true for certain symbolic inputs (Lyons and Beilock, 2013; see also Fig. 2). Instead, albeit speculative, symbols may provide direct retrieval to a richer network of associative links (for similar theoretical positions, see Deacon, 1997; Nieder, 2009; Wiese, 2003).[c] It may also be the case that, at least for overlearned ordinal relations, this direct retrieval access to ordinal information may trump magnitude-based mechanisms of assessing order (Logan and Cowan, 1984; see Franklin et al., 2009, for a similar suggestion). This in turn may help explain why the reversal of distance effects in symbolic ordinal tasks depends on whether the stimuli are in fact in the correct order. Those that are in order permit direct retrieval of ordinal information (and adjacent items are retrieved faster); whereas those that are not in order must instead be processed in a magnitude-based manner (thus engendering a classical distance effect in such cases). Nonsymbolic magnitudes, by contrast, do not have access to ordinal associations, implying the presence of canonical distance effects in all contexts (which is precisely what was found).

### 2.4 SUMMARY

In sum, multiple sources of neural and behavioral evidence support the need to distinguish between ordinal and cardinal processing of numbers. Importantly, however, various contextual factors—in particular whether quantities are presented symbolically or nonsymbolically—may bias the degree of this dissociation. In the next section, we turn to the question of whether ordinal processing of numbers is distinct from ordinal processing of other, nonnumerical stimuli that can be ordered.

## 3 IS NUMERICAL ORDER SPECIAL?

Numbers share important ordinal properties with nonnumerical categories such as letters of the alphabet, days of the week, months of the year, and so on. In this section, we review a small but growing body of evidence—both neural and behavioral—indicating important similarities between numerical and nonnumerical ordinal processing. Despite these similarities, it remains unclear whether these similarities are driven by common processes or representations (an idea we also return to in a later section). In general, precisely how numerical order fits into the broader range of ordinal capacities remains relatively understudied and so provides ample opportunity for future research.

[c] The reader might also find it useful to skip ahead to Fig. 3A for a visual illustration of the various types of ordinal associations.

### 3.1 SPECIFICITY OF NUMERICAL ORDER IN THE BRAIN

As described in a previous section, patient CO showed not only difficulties in accessing the ordinal meaning of numbers, but also exhibited deficits with other nonnumerical symbolic sequences (Turconi and Seron, 2002). For instance, CO was unable to decide if a presented letter came before or after the letter M, whether a day came before or after Wednesday, and whether a month of the year came before or after June. This common pattern of deficits indicates a similarity in the neural organization of numerical and nonnumerical ordered sequences.

One of the first neuroimaging studies to investigate the extent to which the neural correlates associated with numerical and nonnumerical order are similar or different was conducted by Fulbright et al. (2003). In this fMRI study, participants were asked to judge whether three letters, (symbolic) numbers, or arbitrary shapes were in order (ascending or descending) or not (some other permutation). Control tasks were corresponding identity judgments (letters, numbers, shapes, respectively). After subtracting activity from the respective control conditions, similar brain networks were found for letter and number ordinal conditions. Activation overlap for numbers and letters was primarily found in parietal, prefrontal, premotor, occipital, and basal ganglia regions. The common brain activity for numbers and letters provided initial evidence that numerical and nonnumerical ordinal processing engage similar brain regions, and that computational mechanisms may be shared across different classes of ordinal stimuli. It should be noted, however, that the functional overlap in this study was not statistically tested, but rather inferred from visual inspections.

In another study probing the neural correlates of both numerical and nonnumerical order, Ischebeck et al. (2008) found a similar result. Participants silently recited numbers from 1 to 12 and months of the year from January to December (Ischebeck et al., 2008). Relative to the categorical (ie, nonordinal) control condition, the authors found common brain activation in left premotor, prefrontal, and bilateral parietal regions. This activation pattern provides further evidence for the involvement of similar brain regions when ordinal relationships in numerical and nonnumerical stimuli are processed, and it demonstrates that the result generalizes across different experimental paradigms.

Fias et al. (2007) provided still more converging evidence for similar neural correlates underlying numerical and nonnumerical ordinal processing. They used a two-item comparison paradigm with number (symbolic), letter, or saturation (akin to luminance) stimuli. Specifically, participants were instructed to decide which of two presented numerals was numerically larger, which of two letters came later in the alphabet, and which of two squares was more saturated (the lattermost was treated as the control condition). A conjunction analysis revealed the engagement (activity jointly higher than control) of a highly similar brain network for processing letters and numbers, comprising regions of the occipital, temporal, frontal, and parietal cortices. Interestingly, Zorzi et al. (2011) subsequently reanalyzed a portion of the Fias et al. data and came to a somewhat different conclusion. Given its generally recognized importance in number processing, Zorzi and colleagues focused specifically on the overlapping IPS regions found for the letter and number comparison tasks (the other overlap regions from Fias et al. were not analyzed). Using a multivoxel pattern analysis approach, they were able to successfully classify (ie, distinguish between) number and letter trials. Hence, while overlapping brain regions were found for these tasks in the univariate analysis presented by Fias et al., this overlap masked a more fine-grained distinction in terms of the voxelwise response patterns each stimulus-type elicited (at least within the IPS; we also return to this apparently contradictory result in a later section on ordinal mechanisms).

Thus, the three neuroimaging studies discussed above (Fias et al., 2007; Fulbright et al., 2003; Ischebeck et al., 2008) converge to indicate that common brain networks are activated for both numerical and nonnumerical types of ordinal processing (though Zorzi et al., 2011, provide an important cautionary note). It is also worth noting that in each case, the authors primarily focused on symbolic stimuli, and indeed Fias et al. (2007) conducted their conjunction analysis by expressly subtracting the nonsymbolic (saturation) condition from the two symbolic conditions (letters and numbers). We have already noted how symbolic and nonsymbolic numerical ordinal processing differ in important ways (eg, Lyons and Beilock, 2013). Hence, an important and largely unaddressed issue concerns whether similar neural responses for numerical and nonnumerical ordinal processing are found for *nonsymbolic* stimuli.

Furthermore, the authors of the papers reviewed above focused their discussion primarily on the parietal cortex, and the IPS in particular. That is, both theoretical discussions and region of interest (ROI) analyses (in Fias et al., 2007; Ischebeck et al., 2008; Zorzi et al., 2011) tended to focus primarily on the IPS. This is understandable given the high degree of attention that has been paid to the parietal cortex and the IPS in particular in the numerical cognition literature (for a review, see, eg, Ansari, 2008; Dehaene et al., 2003; Nieder and Dehaene, 2009). And indeed, the fact that the authors' results largely generalized to other, nonnumerical types of stimuli calls into question claims about the specificity of the IPS with respect to numerical processing. On the other hand, a high level of preoccupation with one particular brain region may blind researchers to other interesting patterns in the data. For instance, Fulbright et al. (2003), Fias et al. (2007), and Ischebeck et al. (2008) all found common ordinal processing of (symbolic) numerical and nonnumerical stimuli not just in parietal, but also premotor and prefrontal cortices (though see Footnote d for an interesting exception in the case of Zorzi et al., 2011).

Consistent with the notion that ordinal processing is not restricted to the IPS, Lyons and Beilock (2013) found neural activity specific to ordinal processing of number symbols[d] in premotor cortices, including dorsal and ventral left lateral

[d]Areas were localized based on the contrast of symbolic number ordering greater than luminance-ordering control. Left PMd and PMv each showed greater brain activity for symbolic ordinal processing than symbolic cardinal processing (numerals), and nonsymbolic numerical ordinal and cardinal processing (dot arrays). Pre-SMA showed the same result, with the exception that symbolic ordering was not significantly greater than nonsymbolic ordering (Lyons and Beilock, 2013, Table 5). Note that this latter result is broadly consistent with Zorzi et al. (2011), who found that multivariate classifiers were unable to classify numerical and nonnumerical symbolic ordering in SMA, suggesting SMA may process ordinality in a highly general manner. Zorzi et al. did not examine the other premotor regions from Fias et al. (2007).

premotor areas (PMd and PMv, respectively) and presupplementary motor area (pre-SMA) (Fig. 2B). Premotor cortex may be of particular interest with respect to ordinal processing as this region also shows greater activity for extraction of numerical information from tone sequences in both humans and monkeys (Wang et al., 2015), and it is richly populated with order-sensitive neurons in general (Berdyyeva and Olson, 2010). Thus, just as the majority of prior work in the area of numerical cognition has focused on cardinal processing of numbers, much of this work to date has also tended to focus on the parietal lobe. As this review has illustrated, it is important to consider the role of ordinality as well as cardinality in how we understand numbers; hence, it is perhaps also reasonable that we extend our focus to number-relevant brain regions beyond parietal cortex. In this respect, both Lyons and Beilock (2013), as well as several of the studies discussed earlier, converge to indicate that premotor areas may also be key to understanding the ordinal side of (at least symbolic) numbers.

Potentially consistent with a more domain-general view of numerical ordinal processing, premotor cortex has been associated with a wide range of potentially relevant processes that are not strictly numerical. For instance, the SMA has been shown to be important for sequential order processing more broadly (Gerloff et al., 1997; Tanji, 2001), and PMd and PMv areas are involved in retrieval of action plans in response to overlearned symbolic associations in a wide variety of contexts (Grafton et al., 1998; Hoshi and Tanji, 2007; O'Shea et al., 2007; Wise and Murray, 2000). Of course, precisely what these brain areas may mean in terms of understanding the mechanisms behind acquisition and access of ordinal information in number symbols remains an open area of research. However, considering such mechanisms from other cognitive domains underscores the general theme of this section that ordinal processing of number very likely draws substantially on more general mechanisms for processing ordinal stimuli. We come back to this topic in greater detail in a subsequent section by examining various cognitive mechanisms that may be associated with ordinal processing.

### 3.2 HOW NUMBER SPECIFIC ARE CANONICAL AND REVERSE DISTANCE EFFECTS?

In the previous section, we reviewed evidence that distance effects—specifically whether they are canonical or reversed—can distinguish between ordinal and cardinal processing of (symbolic) numbers, as well as between symbolic and nonsymbolic ordinal processing of numbers. It is well known that canonical distance effects are not unique to numerical stimuli,[e] so it seems useful to consider whether behavioral signatures of numerical ordinal processing also generalize to nonnumerical sequences. It is also worth noting that, to the best of our knowledge, this question has been asked almost exclusively of symbolic stimuli (eg, letters, months, weeks,

[e]For example, distance effects are found when fruit flies discriminate between odors (Parnas et al., 2013), and when humans discriminate between species using abstract line drawings of animal figures (Gilbert et al., 2008).

etc.). An important but still unexplored question concerns whether the behavioral signatures of nonsymbolic numerical ordinal processing generalize and/or relate to other types of nonsymbolic ordinal processing (eg, luminance, size, length, etc.).

Using pairwise comparisons, (which of two stimuli comes later in a given ordered set), several researchers have demonstrated canonical distance effects for nonnumerical stimuli. The presence of a canonical distance effect in letter comparisons was first reported in a study by Hamilton and Sanford (1978). In this study, participants were asked to indicate whether or not letter pairs were presented in the correct alphabetical order (ie, ascending) or not (ie, descending). Similar canonical distance effects have also been reported for days of the week (Gevers et al., 2004), months of the year (Gevers et al., 2003), and even after human adults were trained to learn arbitrary sequences with novel symbols (van Opstal et al., 2008, 2009).[f] On the other hand, the extent to which numerical and nonnumerical distance effects are directly related to one another remains largely unknown. To the best of our knowledge, only one study has thus far reported a correlation between the size of the distance effect of numbers and letters (Attout et al., 2014), possibly indicating an association between the processing of numerical and nonnumerical sequences. However, this correlation was eliminated once the reaction times of a luminance discrimination task were taken into account, indicating that the observed correlation may be explained by shared domain-general mechanisms (eg, response selection) rather than a common representation of numerical and nonnumerical sequences.

Investigations that have focused on triplets (which, as noted earlier, tend to be more reliable in eliciting reverse distance effects for symbolic numbers) have revealed the existence of reverse distance effects for nonnumerical order judgments as well. For instance, Franklin et al. (2009) had participants indicate whether triplets of numbers and months were in increasing order or not. Results showed a reverse distance effect for both types of stimuli (numbers and months). Here again, an examination of the relation between reverse distance effects for numbers and months would be ideal. That said, both numbers and months showed larger reverse distance effects on trials where a category boundary was crossed (ie, numbers crossing a decade, months crossing the year boundary), which lends a degree of additional specificity, though admittedly the precise reason for this result is unclear. In general, the current evidence is rather limited and more work is needed to better understand the precise nature of apparent similarities in distance effects for numerical and nonnumerical ordinality judgments. In the next two sections, we turn to the origins of ordinal understanding and the potential mechanisms by which ordinality in numbers may be acquired and processed. In this way, the similarities and differences between symbolic, nonsymbolic, numerical, and nonnumerical ordinal processing may begin to come into better focus.

[f] One may notice that these results appear to differ from Turconi et al. (2006), who reported a reversal of the distance effect for numbers even for pairwise judgments. That said, Turconi et al. found reverse distance effects only when they examined increasing trials, so it is possible that a similar pattern might be found if analyses were similarly constrained with these other nonnumerical stimuli.

### 3.3 SUMMARY

Both neural and behavioral evidence suggest a high degree of similarity for processing numerical and nonnumerical ordinal information. On the other hand, there is currently far too little evidence to conclude that numerical ordinal processing can "merely" be reduced to nonnumerical ordinal processing. For one, it is entirely possible that in certain cases the reduction runs in the other direction, with numerical ordinality providing the underlying mechanism for other types of ordinality. For example, letter ordinality may perhaps be reducible to numerical ordinality. Moreover, even if numerical ordinal processing is reducible to other forms of ordinal processing in other domains (eg, action or verbal sequencing), it is far from clear into which domain or domains numerical ordinality should be subsumed. Finally, as we return to in a later section on mechanisms that support numerical ordinality, there are likely multiple such mechanisms, some of which are more domain general, and others which are more specific to numerical processing. Thus, the right question to ask may not be whether numerical order can be reduced to domain-general processing mechanisms, but instead it may be most fruitful to understand the different facets and levels of ordinal processing, and hence the extent to which each of these may or may not be more general or specific with respect to processing domain. In sum, the relation between numerical and nonnumerical ordinal processing presents both a substantial gap in our understanding, and thus also a major opportunity for future research.

## 4 INCREASING ORDINAL COMPLEXITY: FROM NONHUMAN ANIMALS TO DEVELOPMENT AND ACQUISITION OF ORDINALITY IN HUMANS

As we have seen from previous sections, ordinal processing appears to be highly sensitive to context. In addition, it also allows for a wide range of ordinal relations and inferences. For instance, to know what comes after one-hundred, one does not have to mentally traverse the entire count-list in a step-by-step fashion. Indeed, even in nonhuman primates, understanding of ordinal position allows one to go beyond simple associative chaining. Thus, it appears as though mentally representing relative order allows for a much richer set of information—eg, ordinal position, associative links between nonadjacent items, and relative ordinal direction (Fig. 3). In other words, one of the key features of ordinal processing is that it is highly multifaceted. Moreover, the complexity and richness of ordinal information appear to be gradually acquired over the course of human development. This complexity may also prove especially useful in acquiring associative relations that link abstract symbolic representations of number in particular.

### 4.1 COMPLEX ORDINAL PROCESSING IN NONHUMAN ANIMALS

Terrace et al. (2003) demonstrated that monkeys (*Macaca mulatta*) are capable of understanding ordinal relations beyond simple item–item chaining. Using an innovative "simultaneous chaining" paradigm, Terrace and colleagues (2003; for a

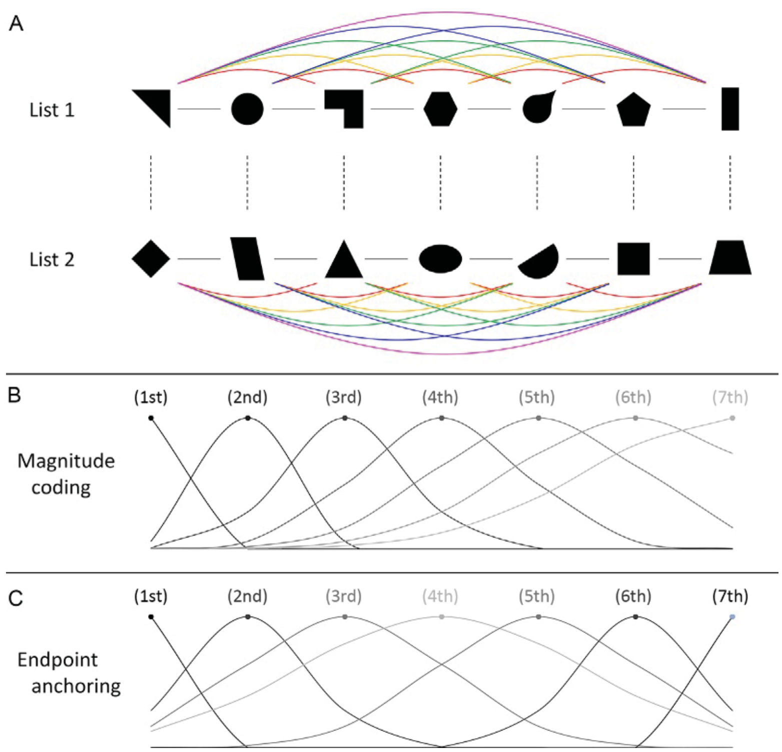


**FIG. 3**

Schematic view of different types of ordinal associations. (A) shows two arbitrary but ordered lists or sequences. Item–item associations between adjacent items within a list are shown with *gray lines*. Deeper associations between nonadjacent items (ie, those that are often inferred associatively) are shown with *lines in rainbow colors*. Associations between lists based on ordinal position are shown with *dashed lines*. (B) Ordinal positional coding based on magnitude. The width of each curve corresponds to the accuracy or precision of a given positional code, with the first-item coded most accurately, and precision decreasing thereafter. (C) Coding scheme based on endpoint anchoring, wherein the endpoints of the list are represented most accurately, with precision decreasing as one moves toward the middle of the list or sequence.

review, see Terrace, 2005) trained monkeys to memorize several 7-item lists. Stimuli were arbitrary images of objects and scenes, and each trial proceeded with the monkey attempting to select (using a touchscreen) the entire sequence in order. The spatial location of each image was randomized on each trial to prevent simple memorization of motor or spatial sequences. In addition, the only form of feedback was that the trial would terminate if an incorrect image was selected—ie, if the selected image was not in fact the next in the sequence. Reward was not given until the entire sequence was produced correctly. Negative feedback (in the form of trial termination) was thus not item specific, and it was accompanied by a several second delay before the next trial began. This meant that memorization of current ordinal position was crucial for making use of any information provided by an incorrect response. For instance, the knowledge that image *X* is incorrect with the first two images already known is applicable only to identifying the third image in the sequence (*X* could still be the 4th, 5th, 6th, or 7th image). Perhaps most remarkable is that, after all four 7-item lists were correctly learned, monkeys were able to substitute images from different lists at their correct ordinal position. If a monkey was presented with a mixture of images from the four lists, they could order the new list based on each image's ordinal position in the list from which it originated (conflicting ordinal positions were avoided in the novel lists). Indeed, when seeing a novel, mixed list, monkeys were able to correctly produce the entire 7-item sequence 91% of the time on the first trial (ie, *dashed lines* in Fig. 3A). It is difficult to see how such a result could be obtained if monkeys merely memorized each list as a simple chain of direct associations (the first item to the second, the second to the third, and so on). Instead, ordinal information was both positional and abstract in that it could be generalized to a new context.

This is not to say that item–item associations play no role in ordinal processing (*gray lines* in Fig. 3A). However, it is perhaps unclear how even these simple associations are processed. When asked to memorize a list and then given the item that comes after some probe item, response times increase as a function of the position of the item in the list (Sternberg, 1967). One account is that an individual must traverse the item–item associations, thus generating longer response times as the number of items to be traversed increases (for a review, see Marshuetz and Smith, 2006). Using a computational modeling approach, Verguts and Van Opstal (2014) present an interesting alternative account: they showed that much the same positional effect can be obtained by simply manipulating how frequently items are presented so that frequency declines with increasing ordinal position. Indeed, such frequency asymmetry is precisely what one would expect if lists are memorized via a rehearsal strategy, the role of which has long been recognized in generating primacy effects in list-recall paradigms more generally (Rundus, 1971). The important thing is that both accounts highlight the importance of item–item associations. The first one asserts that serial position effects are the result of the number of associations traversed; the other suggests one need not traverse the entire sequence, and that positional effects may instead be due to the frequency (and hence the retrieval efficiency) of a given item. This latter account is more consistent with the importance of encoding positional information in addition to item–item associations. It is also consistent with results obtained in studies on the maintenance of list information in short-term memory. When multiple sublists are held in memory, one is likely to confuse items in the same relative position within each sublist (Henson, 1999).

While it is clear from the above that item–item associations play a role in ordinal processing, humans and other species are also capable of inferring nonadjacent ordinality even when trained only with respect to adjacent item–item information. For instance, when trained on $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$, $D \rightarrow E$, humans, primates, and even pigeons can infer that $A \rightarrow C$, $B \rightarrow E$, and so on (eg, Treichler and

Van Tilburg, 1996; Van Opstal et al., 2008, 2009; von Fersen et al., 1991). Thus, ordinal processing goes beyond adjacent item–item associations not only in terms of ordinal position, but also allows for a richer network even of item–item associations (*rainbow-colored lines* in Fig. 3A).

Against this backdrop, numerical and other types of magnitude (eg, size, luminance, length, etc.) are particularly interesting because in one sense they can be thought of as providing inherent cues to relative order. That is, the majority of studies noted earlier relied on lists whose relative order was entirely arbitrary with respect to the properties of the stimuli themselves; hence, being arbitrarily defined, relative order had to be learned. Two stimuli of different magnitudes (eg, a small circle and a large circle, an array of a few dots and an array of many dots, etc.) inherently allow for a magnitude-based distinction that can in turn be used to construct an ordinal sequence. To this end, Brannon and Terrace (1998) trained monkeys to respond to arrays of dots in terms of increasing numbers of dots in each array ($1 \rightarrow 2 \rightarrow 3 \rightarrow 4$). Monkeys were then shown sets of arrays containing 5–9 dots, and they were capable of responding in ascending order to the novel set. In essence, the monkeys learned to recognize relative ordinal information in nonsymbolic magnitudes and were able to transfer this to a new set of magnitudes. Brannon (2002) also showed that human children as early as 9 months are sensitive to ordinal direction in relative perceptual magnitudes.[g]

Interestingly, at least in monkeys, Brannon et al. (2006) demonstrated that sensitivity to global ordinal direction is dependent upon reference points. Monkeys were trained either on ascending ($4 \rightarrow 5 \rightarrow 6$) or on descending ($6 \rightarrow 5 \rightarrow 4$) sequences of nonsymbolic magnitudes and were then tested on pairs of numbers (all combinations 1–9). The monkeys' task at test was to order the pair in the same direction as training (eg, $2 \rightarrow 3$ for ascending training and $3 \rightarrow 2$ for descending training). Critical test pairs involved only magnitudes not included in training (1–3, 7–9). Results showed that, regardless of training order, monkeys were not able to work backward from the starting reference point of the training set (down from 4 in the ascending condition, up from 6 in the descending condition). Specifically, monkeys were above chance so long as at least one of the magnitudes in the novel test pair was in the direction implied by the training set (7–9 for ascending, 1–3 for descending). Monkeys were at or below chance when both items in a test pair were in the direction opposite to that implied by training (1–3 for ascending, 7–9 for descending). The relative difference inherent when comparing perceptual magnitudes *can* be used to infer relative order; however, it appears that this capacity is highly dependent upon context—for instance upon ordinal direction and the reference point of the training set. Note that this conclusion is in keeping with the review of ordinality and cardinality above: the two are certainly intertwined to some extent, but it would be a mistake to consider them one and the same.

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[g]Note that Suanda et al. (2008) later demonstrated that children at 9mos require multiple converging cues (eg, individual item size, overall surface area, etc.); whereas by 11mos, both Suanda et al. and Brannon (2002) showed that children's sensitivity to ordinal direction of nonsymbolic magnitudes is more robust to incongruent cues as well.

## 4.2 GOING BEYOND SIMPLE ITEM–ITEM ORDINAL ASSOCIATIONS IN HUMAN DEVELOPMENT AND LEARNING

Evidence from early acquisition of the meaning of number words in human children also coincides with the work reviewed earlier, by showing that ordinal understanding can (and often does) go beyond simple item–item chaining. However, this deeper understanding of ordinality develops only gradually. Children around the age of 2 or 3 are often able to recite the verbal count-list (one, two, three) in the correct order, and they even understand that each word refers to a distinct number. Crucially, however, they are often unaware to which number each word refers (Wynn, 1992). Young children, in other words, are able to recite an ordered chain of words prior to cardinal understanding (ie, knowing how the chain of words can be applied to counting a set of objects such that the last word said indicates the cardinal value of the set). Colomé and Noël (2012) demonstrated that children 3–5 years are better at solving numerical tasks when phrased in cardinal (How many) vs ordinal (What position) terms. This may be due in part to the fact that cardinal number words (one, two, three) are encountered more frequently than ordinal number words ("first, second, third"; Dehaene and Mehler, 1992). On the other hand, Michie (1985) showed that children 3–5 years tend to be able to match visually presented sets of items in terms of their cardinality prior to being able to assess whether sets of items are correctly ordered. Similarly, Knudsen et al. (2015) recently demonstrated that in children 4–7 years, the ability to sort sets of numerals into the correct order lags behind verbal (number–word) cardinal understanding. However, cardinal and ordinal understanding of *numerals* appeared to develop *concurrently* between the ages of 4 and 5. Together, the above evidence seems to indicate that cardinal understanding of number-words and sets of visual objects precedes ordinal understanding thereof, though the impact of word-frequency and whether this pattern extends to the acquisition of numerals remain unclear. Regardless, these results suggest that simply being able to recite the count-list does not imply either cardinal or more sophisticated ordinal processing. That said, the precise developmental relation between and trajectories of cardinal and ordinal understanding of numbers remains somewhat unclear. What perhaps can be said most clearly is that—at least compared to cardinality—there is a relative dearth of research on the early acquisition of ordinal understanding and processing of numbers in human children. Such work might prove especially fruitful in understanding the acquisition and efficient processing of numerical symbols.

Results from adult training studies may shed some light on the contribution of ordinality and ordinal inferences to the acquisition of numerical symbols. Lyons and Beilock (2009) trained adult participants to associate approximate quantities of dots (presented too quickly to count) with a novel set of abstract figures. Each figure was repeatedly paired with a given quantity. Participants were instructed to learn these pairings as well as they could. Participants were then tested on their ability to perform pairwise numerical judgments using the newly learned set of novel numerical "symbols" (ie, absent any other numerical cues), tested on their ability to reconstruct the symbols' global order (ie, arrange the full set of symbols in increasing order), and finally probed for any strategies they might have employed. Participants who reported using an ordinal strategy[h] performed better not only on the global ordering task, but also on the pairwise numerical tasks. Crucially, these participants performed no better on numerical tasks involving strictly nonsymbolic magnitudes (dot arrays) or overlearned number symbols (Indo-Arabic numerals). In other words, the focus on ordinal information was particularly useful specifically for the *acquisition* of number symbols. Using a similar training paradigm in an fMRI experiment, Lyons and Ansari (2009) demonstrated that a participant's postscan aptitude for reconstructing the symbols' global order was related to greater dissociation (with training) of neural activity in bilateral IPS on tasks that tested numerical comparison vs visual recognition. Furthermore, Merkley et al. (in press) directly contrasted the use of ordinal and cardinal information in learning a novel set of symbols. Half of adult participants were given only ordinal information (the relative order of the symbols), and the other half were given only cardinal information (participants learned to associate an approximate magnitude with a given symbol, presented in random order, similar to Lyons and Beilock earlier). Those given ordinal information significantly outperformed those given cardinal information in a standard comparison task (greater or less than the middle symbol) using the novel symbols. Indeed, using a similar novel-symbol mapping paradigm, Merkley (2015) showed that 6-year-old children could learn to use the symbols in a numerical context (eg, compare which of two is numerically greater) *only* if they were given ordinal in addition to cardinal information. That is children given only cardinal information were at chance on all tasks, whereas those given both ordinal and cardinal information were significantly above chance. Taken together, these results suggest that ordinality may play a key role in our increasing reliance on symbolic representations for understanding and manipulating quantities over the course of development and learning.

Consistent with this idea, Lyons et al. (2014) showed that in Grades 1 and 2, basic cardinal processing of number symbols captured more unique variance in Dutch children's arithmetic scores than any other numerical task (at least among the seven others tested in that study, including ordinal processing). However, the unique variance captured by ordinal processing of number symbols steadily increased such that by Grade 6, it captured significantly more variance than any of the other seven numerical tasks. Consistent with this developmental trend, Vogel et al. (2015) showed that, in Grade 1 children, distance effects from a numerical comparison task predicted arithmetic scores, but distance effects from a numerical ordering task did not. Examining a set of adult participants, Lyons and Beilock (2011) showed that including symbolic numerical ordering performance (a combined measure of response times and error rates) in a regression model completely accounted for the

[h]Interestingly, these participants also tended to be higher in WM capacity. Examining the other end of the spectrum, recent work indicates that deficits in WM for serial-order information may underlie numerical deficits—such as developmental dyscalculia—more broadly (Attout and Majerus, 2015; Attout et al., 2015). We return to the topic of WM in ordinal processing in the next section.

variance in complex mental arithmetic scores captured by both symbolic and nonsymbolic numerical comparison tasks (better ordering predicted better arithmetic performance). However, Goffin and Ansari (2016) showed that symbolic ordering and comparison distance effects in adults each captured unique variance in arithmetic scores. This suggests that cardinal processing of number symbols may retain an important role in more sophisticated numerical skills even with the growing role of ordinal processing. Interestingly, both Vogel et al. (2015) and Goffin and Ansari (2016) showed no relation between symbolic ordering and comparison distance effects. As noted in an earlier section, this underscores a likely dissociation between ordinal and cardinal processing, at least for symbolic numbers. In addition, both this result and the changing contribution of ordinality to more complex math skills over the course of development point to the multifaceted nature of number symbols (Delazer and Butterworth, 1997).

### 4.3 SUMMARY

In sum, as we have argued throughout this paper, overlooking the contribution of ordinality to how we process numbers is likely a major oversight. In a similar vein, the data reviewed in this section also make it increasingly clear that ordinality is not a unitary concept, but involves multiple representations, many of which go beyond simple item–item associations, and appears to increase in relational complexity over the course of developmental time. On the other hand, it is not entirely clear how each of the different types of (especially numerical) ordinal associations is in fact processed, providing a clear opportunity for future research. In the next section, we turn to the various mechanisms by which ordinality is processed, and how these may change over development as well.

## 5 MECHANISMS THAT SUPPORT NUMERICAL ORDINAL PROCESSING

Thus far, we have reviewed how ordinality differs from cardinality, to what extent numerical is distinct from nonnumerical ordinal processing, and how the multifaceted nature of ordinality emerges with development and learning. In the following section, we examine several mechanisms that contribute to the various forms of numerical ordinal processing (eg, symbolic vs nonsymbolic). We begin by returning to the notion that magnitude or cardinal mechanisms play a role in certain types of ordinal processing. Next, we review a growing body of literature examining working memory (WM) for serial-order information, which suggests that this capacity is not necessarily specific to numerical information. We also examine how spatial mechanisms may interact with ordinal processing, especially in WM. Finally, we examine more long-term mechanisms based on ordinal associations that may be especially crucial for understanding how we process ordinality of number symbols. We argue that the more general short-term mechanisms for processing order information interact with long-term memory networks that are more specific to numbers—especially in the case of associative connections between number symbols. In sum, however, the mechanisms for numerical ordinal processing remain relatively underspecified, which provides ample opportunity for future research.

## 5.1 MAGNITUDE-BASED MECHANISMS

In an influential review, Marshuetz and Smith (2006) suggested that both magnitude and associative (or retrieval-based) mechanisms play a role in ordinal processing. Consistent with the notion that magnitude- or cardinal-based mechanisms play a role in ordinal processing, we saw in a previous section on the distinction between ordinal and cardinal processing that, while ordinal processing is unlikely to be reducible to cardinal processing (especially in the case of number symbols), there are contexts in which the two are closely intertwined. Botvinick and Watanabe (2007) formalized the hypothetical connection between ordinal position and magnitude in a computational model in which representation of ordinal position in prefrontal cortex was based on the conjunction of item and approximate magnitude information, suggesting that ordinal position is derived at least in part from approximate magnitude representation. One key feature of the model was that it relied on the assumption of compressive magnitude scaling (decreasing precision as magnitude increased—a key signature of nonsymbolic magnitude processing across species; Nieder, 2005; see also Nieder and Dehaene, 2009). Consistent with this notion, Petrazzini et al. (2015) recently found that guppies (*Poecilia reticulata*) are sensitive to ordinal position independent of (and may even supersede) spatial position. Performance when the critical item was in the 5th position was worse than when it was in the 3rd position, suggesting a degradation in representation akin to compressive magnitude scaling. Furthermore, Ninokura et al. (2003, 2004) demonstrated that the presence of neurons tuned to specific serial ordinal positions in lateral prefrontal cortex are highly reminiscent of neurons that have been found to be tuned to nonsymbolic magnitudes in prefrontal and parietal cortices (for a review, see Nieder, 2005; an idealized version of these curves can be seen in Fig. 3B).

One possibility is that a magnitude-based mechanism of assessing relative order is the more general process, allowing for the broadest range of inputs. That is, unlike the acquisition of symbolic associations, no special training or learning is required to compare the relative magnitudes of various stimuli and infer their relative order therefrom. In other words, one would expect the magnitude-based system of assessing order to be quite general, in that it would apply to both nonsymbolic as well as unfamiliar symbolic inputs (whereas processing of symbolic inputs might show idiosyncrasies specific to a given class on inputs—letters, numerals, months, etc.). This view would account for certain disparities in the literature—such as the contextually dependent nature of reverse distance effects and the seemingly contradictory findings from Fias et al. (2007) and Zorzi et al. (2011). Moreover, as noted in a previous section, Attout et al. (2014) found significant zero-order intercorrelations between distance effects in letter and number ordinal judgments; but this relation was eliminated after controlling for luminance distance effects. Perhaps, these distance effects were more indicative of a general short-term mechanism and less of a direct overlap between long-term ordinal representations of letters and numbers.

Regardless, such an account must for the moment remain speculative. Future work might test the notion by further examining in greater detail the interrelations between different types of nonsymbolic ordinal stimuli (eg, luminance, size, dot arrays, etc.) in terms of their various behavioral signatures, as well as distributed patterns of neural activity, and so forth. In the next section, we turn to another general mechanism (ie, one not specific to numerical or even magnitude inputs) that plays an important role in how we process and maintain ordinal information—serial-order WM.

## 5.2 SERIAL-ORDER WM

When holding a set of items in mind (eg, letters), it is generally recognized that memory for the relative order of the items (order working memory, OWM) is dissociable from memory for the items themselves (item working memory, IWM; eg, McElree and Dosher, 1993; Sternberg, 1966; for a review, see Marshuetz, 2005). Broadly speaking, damage to frontal areas has been shown to compromise OWM, whereas damage to temporal areas tends to compromise IWM (Kesner et al., 1994; Milner, 1971). Accordingly, OWM tasks tend to activate prefrontal and parietal areas more so than IWM tasks, and the latter tend to show greater activity in superior and inferior temporal areas (Majerus et al., 2006; Marshuetz et al., 2000).

WM for ordinal information may play a key role in numerical processing. For instance, Attout and Majerus (2015) showed that children with developmental dyscalculia (a persistent deficit in numerical or mathematical processing; note that similar results were also found for adult participants with mathematical difficulties in Attout et al., 2015) performed significantly worse on an OWM but not an IWM task. Furthermore, dyscalculics were significantly slower on a symbolic numerical ordering task than typically developing controls, but not on standard numerical comparison tasks. Finally, OWM scores tended to correlate more strongly with a variety of numerical tasks relative to IWM scores. Taken together, these results suggest that WM—and in particular WM for ordinal information—may play a key role in determining who is more likely to fail vs succeed in acquiring critical numerical skills. Consistent with this interpretation, in the previous section we saw that individuals who reported using an ordinal strategy were more adept at learning to use a novel set of symbols in numerical contexts (Lyons and Beilock, 2009). As it turns out, these individuals also tended to be higher in WM capacity.[i] In other words, those higher in WM were more likely to adopt an ordinal strategy, which in turn was related to more accurate acquisition of the numerical meaning of a set of novel symbols.

[i]OWM and IWM were not distinguished in that study. However, the critical outcome WM measure (based on reading and operation-span tasks; Unsworth et al., 2005) reflected the accuracy with which participants could recall letters in a specific order (under varying dual task conditions).

At the neural level, overlap between numerical order judgments and WM for order has also been found. Specifically, Attout et al. (2014) showed similar distance effects for a letter comparison task (which comes later in the alphabet), a symbolic numerical order task (does a given number come before/after a standard held in mind), and a serial-order short-term memory task (is a two-item probe in the same order as the same two items from a larger set held in WM). Furthermore, overlapping distance effects for all three tasks were observed in the left IPS. This result is broadly consistent with Fias et al. (2007), who showed common activation in bilateral IPS for symbolic number and letter comparison tasks. In other words, the short-term mechanism used for processing numerical order may be the same as that used for processing serial order more generally. On the other hand, as noted in a previous section, Zorzi et al. (2011) reanalyzed the Fias et al. results using a distributed pattern analysis approach and found that the multivoxel patterns associated with letter and number processing within the IPS could be successfully distinguished. One possibility is that there is a similar online or short-term process for ordinal information that is general to the type of input (which might also account for similar distance effects in certain contexts). However, at least for overlearned types of input (eg, letters and numbers), a common short-term process may interact with distinct long-term memory representations.

## 5.3 SPATIAL MECHANISMS

A related mechanism worth considering is the possibility that one might be able to visualize a limited amount of sequential information in a spatial configuration—for instance with earlier items on the left and later items on the right. There is evidence to indicate the presence of systematic spatial biases when processing ordinal information (Gevers et al., 2003, 2004). For example, individuals tend to respond faster with their left hand to items that come earlier in an ordinal sequence (eg, months, days of the week, numbers, and letters) and faster with their right hand to items that come later in the sequence. Moreover, these biases may arise due to spatial organization of both short-term ordinal processing (ie, serial-order WM; Abrahamse et al., 2014; De Belder et al., 2015; Vandierendonck, 2015; van Dijck et al., 2013, 2014, 2015), as well as how we represent ordinal information in long-term memory (von Hecker et al., 2015)—with recent evidence indicating the two sources of spatial bias are dissociable; Ginsburg and Gevers, 2015). Spatial biases are even evident after ordinal training in humans with a novel set of arbitrary symbols (Van Opstal et al., 2009). Indeed, as noted in the previous section, when both humans and other species are trained to learn an arbitrary ordinal sequence by means of feedback on adjacent pairwise comparisons (eg, A → B, B → C, C → D, D → E), one routinely sees transfer to nonadjacent pairs (eg, A → C, B → E; Treichler and Van Tilburg, 1996; Van Opstal et al., 2008, 2009; von Fersen et al., 1991). In humans, this associative transfer in ordinal learning is associated with changes in hippocampal and left angular gyrus activity—both regions associated with memory as well as spatial processing (Seghier, 2013; Vann and Albasser, 2011).

These results are broadly consistent with the notion that ordinal associations provide a basic framework for tapping a more complex web of associations (Lyons and Ansari, 2015; Nieder, 2009). Here we suggest that the ability to spatially visualize ordinal structure may facilitate this process. Consistent with this notion, Lyons and Ansari (2009) showed that, following training, those better able to reproduce the global ordinal structure in a spatial layout were more likely to show increasing dissociation between visual recognition and numerical processing of a novel set of symbols in bilateral IPS. Moreover, as noted previously, Lyons and Beilock (2009) use a similar training paradigm to show that those relying on an ordinal strategy (and higher in WM) were more adept at learning to use the novel symbols in numerical contexts—including the ability to spatially arrange the newly learned symbols in terms of their global ordinal structure. Interestingly, those who succeeded most at this latter task showed an "outside-in" pattern of performance, with the highest accuracy at the endpoints and the lowest accuracy in the middle points (for a similar result in children, see Merkley, 2015). Such a pattern is similar to what one finds in multiitem ordinal comparisons (eg, Jou, 1997, 2003). For instance, one is shown a horizontal array of three numbers presented in a random order; on a given trial, one is asked to identify the location of, say, the "smallest" number. On the other trials, one is asked to find the "middle" or "largest" number. Note that one must first determine the global order of the set to determine which number matches the target criterion (a new random set is generated for each trial) to complete the task. Regardless of spatial position on the screen, one tends to find the "endpoints" (the smallest or the largest numbers) fastest, and search times systematically increase as one progresses inward, with the longest response times arising for the middle number (similar results are found for 5-item sets, and for sets involving letters).[j] Fig. 3C visualizes this overrepresentation of the reference or endpoints in an ordered sequence. This suggests the importance of identifying boundary conditions or anchor points to better structure one's ordinal search process (Brannon et al., 2006; Trabasso and Riley, 1975). Given the intervening gaps, it would seem imprudent to use a verbal rehearsal or direct item–item chaining to represent one's ordinal representation when constructed in such an outside-in manner.

Instead, a spatial representation that allows for simultaneous representation of endpoints with the possibility of filling in interior locations would seem more efficient (Trabasso and Riley, 1975). The outside-in pattern observed for participants who relied on ordinal strategies in Lyons and Beilock (2009) suggests they were constructing a representation of the numerical meanings of the novel-symbol set in a similar, perhaps visuospatial manner. Note also that these participants tended to be higher in WM capacity as well. Taken together with the preceding paragraph,

[j] A similar effect is found when participants are asked to rank various categories in terms of a specific dimension (e.g., actors by age, animals by weight, countries by area, etc.). Performance is most accurate for items at the endpoints (oldest/youngest, lightest/heaviest, largest/smallest, respectively; Kelley et al., 2015). This is consistent with the notion of a more general mechanism—ie, one that is not specific to numbers—as was also suggested in a previous section on whether numerical ordinality is "special."

it seems that ordinal processing both in long- and short-term memory may be spatially organized. The ability to visualize the ordinal structure of multiple items at once may also play a key role in constructing and inferring deeper associations between items, which may be key to understanding how we represent abstract number symbols (Lyons and Ansari, 2015; Nieder, 2009).

## 5.4 THE MECHANISMS UNDERLYING ACQUISITION AND ACCESS OF ORDINAL ASSOCIATIONS

For overlearned symbolic representations with a strong ordinal component, one would expect to find distinct associative networks for different types of symbolic stimuli, such as letters, numbers, etc. These long-term associations might facilitate access to positional information, and so trump other slower, if more general, mechanisms of assessing order (Logan and Cowan, 1984). As has been discussed earlier, classical distance effects are typically reversed when making ordinal judgments over correctly ordered number symbols (eg, one is faster to verify that 4-5-6 is in order than 3-5-7; Franklin and Jonides, 2009; Franklin et al., 2009; Goffin and Ansari, 2016; Lyons and Beilock, 2013; Turconi et al., 2006). Moreover, rapid access to ordinal associations is present even in children who are only just starting their formal math education and persists in a consistent manner thereafter. Lyons and Ansari (2015) showed that children as young as 1st grade also show a reverse distance effect, and that the magnitude of this effect remains relatively stable over at least grades 1–6. An important question, then, is what drives the acquisition of these ordinal associations in numbers?

One possible explanation is that these ordinal associations are the product of highly routinized rehearsal of the count sequence. 4-5-6 is better rehearsed than 3-5-7, so one is faster and more accurate when verifying the former is in order. This is certainly a plausible explanation, though it is worth noting that children can recite the count-list prior to understanding of either the cardinal (Wynn, 1992) or ordinal (Colomé and Noël, 2012) meanings of those count words. Furthermore, Lyons and Ansari (2015) found no relation between reverse distance effects and counting ability in a sample of nearly 1500 children. The authors also found that the relation between performance on ordinal judgments of items such as 4-5-6 and arithmetic scores could not be accounted for by counting performance. In sum, though the role of counting in numerical development certainly should not be discounted in general, it does not seem to provide the key mechanism for understanding how ordinal associations between numbers symbols are processed.

Another possibility is that, through a variety of circumstances—not just counting—one is likely to be highly familiar with ordered, adjacent integers (such as 4-5-6). This increased familiarity could contribute to more rapid recognition of these items (Saumier and Chertkow, 2002). For instance, LeFevre and Bisanz (1986) found that ordinal verification performance was improved not only for adjacent sequences such as 4-5-6, but also for nonadjacent, but nevertheless highly familiar contexts, such as counting by fives: 5-10-15. In addition, Bourassa (2014) found that adults were faster to verify ordered sequences which completed a simple addition problem (eg, 2-5-7, which could also be interpreted as the valid addition equation: $2+5=7$) than nonordered presentations (eg, 5-7-2, where $5+7=2$ is not a valid equation). On the other hand, ordered arithmetic sequences were not verified faster than ordered nonarithmetic sequences (eg, 3-5-7, 3-5-9), so it is unclear precisely what degree of familiarity was at play. Furthermore, double-digit numbers are encountered far less frequently than single-digit numbers (Dehaene and Mehler, 1992), making the former likely less familiar than the latter. And yet, Lyons and Ansari (2015) found that reverse distance effects were roughly twice as large for double-digit relative to single-digit numbers. This suggests that familiarity alone is perhaps insufficient to explain reverse distance effects. Instead, it may be that ordinal associations tap a deeper, more complex web of associations that underpin the abstract nature of number symbols (Lyons and Ansari, 2015; Nieder, 2009). Regardless, though the earlier discussion can help rule out several possibilities, the precise mechanism(s) by which ordinal associations between number symbols are learned and accessed remain unknown, and are therefore a promising candidate for future research. Furthermore, it is unclear whether these associations are specific to a given class of symbolic inputs—for instance, are the long-term associations for numerals distinct from the networks of ordinal associations that link letters, months, and so on? In the next section, we turn to the potential role that numerical ordinal processing—and in particular acquisition of the ordinal associations among numbers discussed in this section—play in the development of more sophisticated types of numerical processing, such as complex mental arithmetic.

### 5.5 SUMMARY

In keeping with the broader notion that ordinality is multifaceted, it appears that several different mechanisms contribute to how we process numerical ordinality. These range from magnitude- or cardinality-based mechanisms, to WM for serial-order information (which in turn may rely heavily on spatial processes), to more long-term mechanisms based on ordinal associations. We suggest that the more general short-term mechanisms for processing order information interact with long-term memory networks that are more specific to numbers—especially in the case of associative connections between number symbols. In the next section, we extend this idea and examine the extent to which ordinal associations among number symbols may provide a crucial foundation for more sophisticated forms of numerical processing.

## 6 ORDINALITY AND IMPLICATIONS FOR MORE COMPLEX NUMERICAL PROCESSING

In this section, we bring together the different lines of evidence reviewed earlier and consider them in the context of understanding how ordinality may contribute to other, more sophisticated forms of numerical processing. Specifically, we saw that ordinality is a key aspect of how we process and understand numerical quantities both symbolically and nonsymbolically. We also saw that ordinal processing of number symbols may differ in certain contexts where overlearned ordinal associations may be relevant to the task (as in the case of reverse distance effects when making ordinality judgments; Franklin and Jonides, 2009; Franklin et al., 2009; Goffin and Ansari, 2016; Lyons and Beilock, 2013; Turconi et al., 2006). There is also evidence to suggest that symbolic and nonsymbolic representations of number are dissimilar more generally (eg, Lyons et al., 2012), and it is possible that access to ordinal associations is key to understanding this difference (Nieder, 2009). Indeed, we saw that individuals who were either instructed to focus on (Merkley, 2015; Merkley et al., in press) or spontaneously focused on (Lyons and Beilock, 2009) ordinal information acquired the numerical meaning of a novel set of symbols more accurately, and their numerical- and recognition-based processing of the novel symbols was more dissociable (Lyons and Ansari, 2009). Taken together, these results tempt one to hypothesize that ordinal associations among number symbols may provide a crucial foundation for more sophisticated forms of numerical processing. For this to be the case, however, the ordinal processing of number symbols should therefore be predictive of more complex forms of numerical processing such as complex mental arithmetic.

Consistent with this prediction, Knops and Willmes (2014) found overlapping frontoparietal areas for symbolic numerical ordering judgments and mental arithmetic, and the two tasks also showed correlated activity patterns across voxels within these regions. At the behavioral level, Lyons and Beilock (2011) found that adult performance on a basic ordinal verification task (are three numerals in the correct left–right order?) captured about half the variance in a complex mental arithmetic task (involving all four basic arithmetic operations with unfamiliar problems that often required carry/borrow operations over multiple digits). Importantly, the numeral ordering task persisted in capturing about 30% of unique arithmetic variance, even after controlling for WM capacity, as well as nonsymbolic number comparison, symbolic number comparison, letter ordering, and numeral recognition performance (indeed, symbolic number ordering was the only significant unique numerical predictor that remained). Goffin and Ansari (2016) replicated and extended this result. When they examined *overall* performance (a measure combining accuracy and response times), both symbolic number comparison and ordering showed significant zero-order correlations with arithmetic performance, but only ordering captured unique variance (controlling also for visuospatial short-term memory and inhibitory control). When Goffin and Ansari instead examined *distance effects*, both comparison and ordering distance effects (canonical in the former case, reversed in the latter case), each captured unique variance in arithmetic scores, which held even after controlling for overall performance on the two tasks. Interestingly, overall ordering performance also remained a significant predictor. As noted in the previous section, reverse distance effects may index primarily the associative aspect of ordinal processing, so it is interesting to see that both reverse distance effects and overall ordering symbolic numerical performance capture separate sources of arithmetic variance.

This is broadly consistent with a multifaceted view of numerical ordinal processing, and it suggests that these variegated facets may contribute differently to more sophisticated forms of number processing, such as complex mental arithmetic.

Importantly, there also appears to be a critical developmental aspect to the relation between ordinality and arithmetic processing. As noted earlier, both Vogel et al. (2015) and Lyons et al. (2014) found no relation between symbolic ordinal and arithmetic performance in 1st graders; however, even after controlling for symbolic number comparison performance (and six other numerical tasks, as well as reading, nonverbal intelligence, and basic processing speed), Lyons et al. found the strength of this relation steadily increased starting in 2nd grade, such that it was the strongest numerical predictor by grade 6. That said, from these data alone, it is unclear whether this developmental shift is due to a change in how ordinality of number symbols is processed or a change in how children do arithmetic.

To address this issue, Lyons and Ansari (2015) showed that reverse distance effects obtained (and remained relatively consistent) in all grades 1–6. Crucially, reverse distance effects were driven in large part by highly efficient performance on increasing, numerically adjacent ordering trials (eg, 2-3-4, 4-5-6, 6-7-8). In this respect, Lyons and Ansari (2015) found that performance on just the numerically adjacent trials in the stimulus set all but completely accounted for the ordering result in Lyons et al. (2014). That is to say, these trials accounted for more unique variance than any other ordering trial type, and when performance on just these trials was substituted for overall ordering performance in the main regression model from Lyons et al. (2014) results were nearly identical. Note that these results also included the developmental shift discussed earlier. In sum, the increasing, numerically adjacent trials were processed similarly across grades (in terms of reverse distance effects), these trials were driving much of the relation with arithmetic, and this relation changed over development. Together, these results imply it is in fact how children *do arithmetic* that is changing. In particular, older children increasingly rely on retrieval strategies (eg, Imbo and Vandierendonck, 2007, 2008), so it seems plausible that the ordering task is an index of the associations which enable efficient retrieval of numerical relations in an arithmetic context. More broadly, this view is consistent with the notion that more sophisticated numerical processing is increasingly reliant upon a rich semantic network of numerical associations (eg, Fig. 3A). It is perhaps for this reason that one sees an increasingly strong relation between symbolic numerical ordering and arithmetic performance. Indeed, as an extension of this hypothesis, LeFevre and Bisanz (1986; see also Bourassa (2014) for a similar result) showed in adults that performance on patterned but slightly less familiar (eg, 2-4-6, 3-6-9) ordering trials best discriminated between individuals with high and low math skills. Note this is in contrast to children—where the presumably more familiar adjacent patters (eg, 3-4-5, 5-6-7) best predicted children's arithmetic scores. Presumably, it is these deeper, nonadjacent associations that become increasingly relevant as adults begin to deal with ever more sophisticated forms of mathematics.

## 6.1 LIMITATIONS

It is important to point out that much of the evidence discussed earlier is largely based on correlational studies. Park and Brannon (2013, 2014) trained adult participants on a symbolic number ordering task and participants tended to show only marginal gains on a symbolic mental arithmetic task. Hence, the causal role of ordinal processing in the development of arithmetic skills should be treated with caution. More generally, it is crucial to make a distinction between a task meant to measure or be an index of some underlying process, and the process itself. To cure a fever, one does not build a more precise thermometer; and by extension, if one demonstrated that using a more precise thermometer indeed failed to reduce one's fever, it would be rather rash to conclude ambient bodily temperature is irrelevant to one's health. Our assertion here is that one's ability to access ordinal associations between symbolic numbers is an index of a complex network of semantic associations between numbers. Simply having participants complete dozens of multiple ordinal verification trials may be akin to trying to build a more precise thermometer to cure a fever. Instead, we argue, the underlying network of associations needs to be expanded and strengthened. Of course, this conclusion must for the moment remain speculative. Regardless, unpacking the precise contributions of numerical ordinal processing to other, more sophisticated types of numerical and mathematical processing is an area rich with major potential theoretical as well as practical implications.

# 7 CONCLUSIONS

Scientists from several different fields (cognitive science, psychology, neuroscience, and linguistics) have used a variety of methods to study how we represent numerical information (both in symbolic and nonsymbolic formats). The study of numerical and mathematical processing is a thriving field, as is aptly illustrated by the contributions in this volume. This research has led to significant advances in our understanding of how numbers are processed and represented. However, the majority of this research has focused on the "how many" question (understanding how the cardinality of numbers is processed and represented). In contrast to the predominant focus on how the cardinality of numbers is represented and processed, there has been comparatively less work on the representation and processing of numerical order.

This state-of-the-art, however, is changing. In recent years, a growing body of evidence from both behavioral and neuroimaging studies with nonhuman primates, human children, and adults has begun to more systematically investigate numerical order processing from a wide range of different theoretical approaches, using a variety of methodological toolkits. In an effort to summarize and integrate this evidence, the aim of the present review was to provide a comprehensive discussion of what is currently known about the processing and representation of numerical order and what future directions in research on numerical order may be particularly fruitful.

As should be evident from the earlier review, one overarching conclusion is that numerical order processing is complex and multifaceted. Specifically, it is becoming increasingly clear that numerical order processing is, at least partially, distinct from the processing of cardinality: It follows different developmental trajectories, is uniquely associated with individual differences in higher-level numerical and mathematical skills, and is underpinned by different neural mechanisms. This conclusion demonstrates that the study of numerical order processing should be given greater attention. Furthermore, there are clearly relationships between the processing of numerical and nonnumerical order at both behavioral and brain levels of analyses; the mechanisms for processing ordinal sequences in other domains very likely play a role in numerical ordinal processing as well. However, the evidence remains incomplete as to whether numerical ordinal processing is completely reducible to these domain- or stimulus-general processes. Put differently, we do not yet know whether numerical order is "merely" the artifact of other types of ordinal processing but instead possesses unique features. Here, we have suggested answering this question may depend critically on the type of mechanism—for instance short- vs long-term memory—in question. Regardless, it is clear that further research is needed to either identify the more general ordinal processes that contribute to numerical ordering and what, if any, features may distinguish numerical from other types of ordinal processing. Importantly, it is also clear that numerical order processing differs between symbolic and nonsymbolic processing and thereby suggests that numerical order processing may be a key differentiator between the two.

This leads to the suggestion—put forward in the present review—that numerical order allows for symbolic representation of number to be less tied to the cardinalities they represent. Instead, numerical order provides a means by which ordinal representations go beyond item–item association and toward a network of associative links that allow humans to process numbers for which they have no perceptual experience of their cardinal values (eg, a billion). In this way, symbolic numerical order processing may reveal much about how symbolic representations of number have both enabled humans to outstrip the numerical abilities of nonhuman primates, and contributed to the complexity of mathematical thinking.

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# The Mathematical Brain Across the Lifespan

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