

The Relation between Subitizable Symbolic and Non-Symbolic Number Processing over the
Kindergarten School-Year

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Data Availability

The data that support the findings of this study are available from the corresponding author (Ian M. Lyons) upon reasonable request.

RESEARCH HIGHLIGHTS

- Symbolic and non-symbolic processing are more strongly related within the subitizing range in both the fall and spring of the Kindergarten year.
- Bi-directional longitudinal relation between symbolic and non-symbolic for trials specifically within the subitizing range (1-4).
- Unidirectional longitudinal relation between symbolic and non-symbolic (symbolic→non-symbolic) processing for trials outside of the subitizing range.

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ABSTRACT

A long-standing debate in the field of numerical cognition concerns the degree to which symbolic and non-symbolic processing are related over the course of development. Of particular interest is the possibility that this link depends on the range of quantities in question. Behavioral and neuroimaging research with adults suggests that symbolic and non-symbolic quantities may be processed more similarly within, relative to outside of, the subitizing range. However, it remains unclear whether this unique link exists in young children at the outset of formal education. Further, no study has yet taken numerical size into account when investigating the longitudinal influence of these skills. To address these questions, we investigated the relation between symbolic and non-symbolic processing inside versus outside the subitizing range, both cross-sectionally and longitudinally, in 540 kindergarteners. Cross-sectionally, we found a consistently stronger relation between symbolic and non-symbolic number processing within versus outside the subitizing range at both the beginning and end of kindergarten. We also show evidence for a bidirectional relation over the course of kindergarten between formats within the subitizing range, and a unidirectional relation (symbolic→non-symbolic) for quantities outside of the subitizing range. These findings extend current theories on symbolic and non-symbolic magnitude development by suggesting that non-symbolic processing may in fact play a role in the development of symbolic number abilities, but that this influence may be limited to quantities within the subitizing range.

Key words: symbolic magnitude processing, non-symbolic magnitude processing, subitizing, kindergarten

INTRODUCTION

Humans have the capacity to represent magnitudes both non-symbolically (such as arrays of dots or sets of apples) and symbolically (such as number words or Arabic numerals). The suite of abilities for representing and manipulating non-symbolic quantities is often referred to as the approximate magnitude system (AMS) or approximate number system (ANS; in keeping with Núñez, 2017, we adopt the former here). The set of abilities for representing and manipulating symbolic quantities is known as the symbolic number system (SNS). Much research is focused on understanding how these foundational magnitude systems underpin more complex numerical and mathematical abilities (De Smedt et al., 2013; De Smedt et al., 2009; Lyons et al., 2014; Toll et al., 2015) and the degree to which these two systems are linked throughout development has been hotly debated.

Two Systems for Symbolic and Non-Symbolic Quantity Processing

On the one hand, the fact that a system for non-symbolic quantity processing is present from early infancy (Xu & Spelke, 2000; Xu et al., 2005), and also within non-human animals (Cantlon, 2012), has led some to suggest that non-symbolic representations of quantity form the foundation from which culturally derived number symbols are learned (Dehaene, 2007; Feigenson et al., 2013). More specifically, some researchers have proposed that humans acquire the meaning of number symbols by mapping them directly onto their non-symbolic counterparts, thereby forging a direct link between the pre-existing AMS and the SNS by which the two systems become systematically integrated over developmental time (Feigenson et al., 2004; Piazza, 2011). However, a growing body of behavioral and neuroimaging research has begun to challenge the notion of a shared representational system for symbolic and non-symbolic processing (Bluthé et al., 2014; Lyons et al., 2012; Lyons et al., 2015; Lyons et al., 2018; Matejko & Ansari, 2016; Mussolin, 2014; Sasanguie et al., 2014). More specifically, some have suggested that symbolic number skills develop independently from the AMS in childhood (Lyons et al., 2018; Matejko & Ansari, 2016; Mussolin et al., 2014; Sasanguie et al., 2014) and that these two systems remain distinct in adulthood (Lyons et al., 2012).

If the development of the SNS is in fact rooted in the AMS, one would expect to see a strong relation between non-symbolic quantity processing and the acquisition of later symbolic number skills; however, the majority of longitudinal studies has failed to provide evidence that

this is the case (e.g., Lyons et al., 2018; Matejko & Ansari., 2016; Mussolin et al., 2014; Sasanguie et al., 2014). While there is a body of literature demonstrating a link between AMS acuity and later symbolic number knowledge (Chu, VanMarle & Geary, 2015; Chu, VanMarle & Geary, 2016; Elliott et al., 2019; Geary et al., 2018; Shusterman et al., 2016), these studies tend to be limited in their ability to make directional inferences regarding the relation between symbolic and non-symbolic processing. More specifically, while many of these studies are longitudinal in nature (i.e. different measures were administered at different time points), the authors do not implement repeated measures and are therefore unable to control for the pre-existing relation between symbolic and non-symbolic processing at time one. As such, these studies do not provide strong evidence for a relation between non-symbolic skills at time one and *growth* in symbolic number abilities (see Bailey et al., 2018).

One exception is a recent study by Elliott et al. (2019) who utilized a repeated measures design to assess directionality in the relation between symbolic and non-symbolic processing. Overall the authors observed evidence for a bidirectional relation between non-symbolic processing and math achievement, with non-symbolic skills at the beginning of preschool predicting growth in math achievement and vice versa. However, it is important to note that Elliott et al. assessed math achievement using the Test of Early Math Abilities 3 (TEMA-3; Ginsberg & Baroody, 2003), which includes items that tap both informal and formal math skills. The informal items include assessments of both symbolic and *non-symbolic skills* while the formal items are primarily symbolic. Interestingly, prior research has shown that AMS acuity is related to the informal, but not the formal, items on the TEMA-3 (Libertus, Feigenson & Halberda, 2013). Given that Elliott et al. (2019) did not distinguish between formal and informal items in their analysis, it is possible that the observed bi-directional relation may be driven by the inclusion of the informal items. Moreover, inclusion of informal items may help explain the discrepancy between the Elliott et al. results and those in studies focusing on a narrower definition of strictly symbolic skills (Suárez-Pellicioni & Booth, 2018; Lyons et al., 2018; Matejko & Ansari, 2016; Mussolin et al., 2014).

For the purposes of the current study we define symbolic number processing as performance on a task that uses strictly symbolic stimuli and therefore does not require perceptual processing of non-symbolic stimuli. On a practical level, this definition of symbolic number processing also aligns closely with the majority of higher-order math assessments, which

tend to rely exclusively on symbolic representations of quantity. Of the repeated measures studies that follow a similar operational definition of symbolic number processing, to our knowledge, all have identified a *unidirectional* relationship between symbolic and non-symbolic processing, with the former predicting growth in the latter, but not the reverse (Suárez-Pellicioni & Booth, 2018; Lyons et al., 2018; Matejko & Ansari, 2016; Mussolin et al., 2014). For example, Lyons et al. (2018) administered both a symbolic and non-symbolic comparison task to 539 kindergarten children at the beginning and end of the school year. The authors observed that while symbolic number abilities at the beginning of kindergarten predicted growth in non-symbolic processing, non-symbolic abilities did not predict growth in symbolic processing. Evidence of a similar unidirectional relationship between these two systems has been observed in preschoolers (Mussolin et al., 2014) and also first graders (Matejko & Ansari, 2016). Taken together, not only do these findings challenge the notion that the AMS drives the acquisition and further refinement of symbolic number abilities, they in fact suggest the opposite – that symbolic number abilities may drive further development of the AMS.

Overall, the evidence reviewed above converges on the idea that non-symbolic quantity processing plays perhaps only a limited role in the development of symbolic number abilities, but the majority of this work has failed to consider that not all non-symbolic representations of quantity are approximate. More specifically, research has shown that humans have the capacity to form rapid and *exact* representations of non-symbolic quantities between one and four, a process referred to as subitizing (Kaufman et al., 1949). Outside of the subitizing range (1-4), individuals must rely on counting to form exact numerical representations of non-symbolic quantities. It is when counting becomes too strenuous, or is excluded as an option, that estimates of non-symbolic quantities become imprecise (Dehaene, 1992). The fact that subitizable quantities can be represented exactly, rather than approximately, suggests that they may be processed more similarly to their symbolic counterparts than are larger non-symbolic quantities.

Exact Representations of Subitizable Quantities

The hypothesis that subitizable non-symbolic quantities may be processed similarly to symbolic numbers stems from research suggesting that small quantities are subserved via systems that allow for exact representation of quantity such as pattern recognition (Mandler & Shebo, 1982) or parallel individuation (Carey, 2009; Feigenson et al., 2002; Feigenson et al., 2004; Lipton &

Spelke, 2004; Revkin et al., 2008; Uller et al., 1999). In terms of pattern recognition, the limited variation in the degree to which small quantities can be arranged means that they are often arranged in familiar patterns, allowing for the rapid apprehension of the total number of items in the set. Given that systematic representation of a meaningful pattern is a hallmark characteristic of symbols, the fact that subitizable quantities are more prone to pattern recognition than are larger quantities suggests that they may be processed more similarly to symbolic numbers.

Further support for the claim that subitizable quantities may be processed similarly to symbolic numbers stems from research suggesting that subitizing depends on parallel individuation rather than estimation (Carey, 2009; Feigenson et al., 2002; Feigenson et al., 2004; Lipton & Spelke, 2004; Revkin et al., 2008; Uller et al., 1999). Unlike the AMS, which provides imprecise estimates of quantity, the parallel individuation system allows for exact representation of the total number of items in a set – albeit in a highly restricted range, typically from about 1 to 4. Support for the notion of subitizing stems from research suggesting that small (1-4) and large (>4) non-symbolic magnitude processing differentially engage attentional resources, such as eye movements and visual working memory (Ansari et al., 2007, Burr et al., 2010; Egeth et al., 2008; Piazza et al., 2011; Watson et al. 2007). For example, using eye tracking data, Watson et al. (2007) found that eye movements are required for the accurate enumeration of quantities outside of the subitizing range but not for the accurate representation of quantities within the subitizing range. More specifically, the authors found that when the participant's ability to saccade was restricted, the enumeration of larger quantities became slower and less accurate while the enumeration of subitizable quantities remained fast and accurate. Furthermore, Piazza et al. (2011) found that an individual's subitizing limit or capacity was predicted by their visual working memory capacity (i.e., the number of discrete items they could successfully recall after a short delay). Estimation performance on sets of larger quantities outside the subitizing range were unrelated to visual working memory capacity. Together, these data lend support to the claim that smaller quantities can be processed via a separate system that allows for an exact representation of quantity. It is this notion of exact representation within the subitizing range that underlies the hypothesis that subitizable quantities may be processed more similarly to symbolic numbers than larger quantities.

The Relation between Subitizable Quantities and Number Symbols

Overall, the body of research reviewed above suggests that smaller quantities can be represented exactly through parallel individuation while larger quantities are represented approximately through estimation. Given that exact representation is a key property of number symbols (Núñez, 2017), it seems plausible that non-symbolic quantities within the subitizing range may be processed more similarly to symbolic quantities than are non-symbolic quantities outside this range. Consistent with this, evidence from adults has shown that the mixing cost (i.e. increase in reaction time) associated with translating between symbolic and non-symbolic representations of number is significantly smaller for quantities within the subitizing range compared to larger quantities (Lyons et al., 2012). It is easier for adults to translate between the two systems when quantities are small rather than large, suggesting that processing of symbolic numbers may be more similar to that of their non-symbolic counterparts within the subitizing range. This hypothesis is further supported by neuroimaging evidence demonstrating stronger similarity between patterns of neural activity elicited by symbolic and non-symbolic quantities for smaller relative to larger quantities (Lyons & Beilock, 2018). Overall, these findings suggest that, in adults, the link between symbolic and non-symbolic processing may be stronger for subitizable quantities that can be represented exactly within both the AMS and SNS.

While the above work was conducted with adults, it is important to understand whether this unique link between number symbols and subitizable quantities is also observed in children at the outset of formal education. If it is the case that the relation between symbolic and non-symbolic number processing is stronger within the subitizing range in young children, this could have implications for educators as they may want to pay particular attention to this link when scaffolding further number development. Previous work at this stage in development points toward a unidirectional relationship between symbolic and non-symbolic magnitude processing, with symbolic skills predicting growth in non-symbolic processing but not the other way around (Lyons et al., 2018; Matejko & Ansari., 2016; Mussolin et al., 2014). As such, this body of literature converges around the conclusion that the AMS plays little to no role in the development of symbolic number skills. However, this line of research has thus far failed to consider that not all non-symbolic quantity estimations are approximate (Carey, 2009; Feigenson et al., 2002; Feigenson et al., 2004; Lipton & Spelke, 2004; Uller et al., 1999). More specifically, subitizable quantities are thought to be represented exactly through parallel individuation

whereas larger quantities are thought to be represented approximately through estimation (e.g., Revkin et al., 2008; Ansari et al., 2007, Burr et al., 2010; Egeth et al., 2008; Piazza et al., 2011; Watson et al. 2007). Given that numerical symbols reflect exact representations of quantity, the fact that subitizable quantities can be represented exactly suggests that the relation between symbolic and non-symbolic processing may be stronger within the subitizing range.

Current Study

The aim of the current study is to investigate whether the relationship between symbolic and non-symbolic processing differs inside versus outside of the subitizing range, both cross-sectionally and longitudinally, in a large sample of kindergarten children. Given prior research indicating that subitizable quantities may be processed more similarly to symbolic numbers than larger quantities, we hypothesize that symbolic and non-symbolic processing should be more closely related for quantities within, than outside of, the subitizing range. Further, it is plausible that the unidirectional relationship between these two systems (in which symbolic processing predicts growth in non-symbolic processing but not the other way around) observed in prior research is being driven in part by the inclusion of larger numbers (Lyons et al., 2018; Matejko & Ansari, 2016; Mussolin et al., 2014). More specifically, given that both number symbols and subitizable quantities can be considered exact representations, it is plausible that practice with one will promote growth in the other to a similar extent. Therefore, we hypothesize the presence of a significant bi-directional longitudinal relation between these two systems specifically within the subitizing range. We chose to investigate these hypotheses within kindergarteners, as children of this age are just beginning formal education and therefore have yet to develop fluency with symbolic and non-symbolic number skills. As such, this is a key stage of development in which to investigate growth in these skills.

METHODS

It is important to note that the data presented here come from a large, longitudinal data set, a portion of which has been described and reported elsewhere (Lyons et al., 2018). However, the current study addresses a unique set of theoretical questions and analyzes the data in a manner distinct from those reported previously.

Participants

Data were collected from 694 Senior Kindergarten¹ children across 36 schools² within the Toronto District School Board (TDSB). Of these 694 students, 154 were removed due to missing data in one or more conditions of interest at either time point (fall or spring). This resulted in a final sample of 540 children (241 female; 67 not born in Canada). Mean age at the first time point (beginning of the school year) was 5.17 years (range: 4.67-5.77, SD: 0.29). Socio-economic status (SES) was not available at the child level, although, it could be estimated for each school³. Schools were categorized as 0=Low-SES (25%), 1=Medium-Low-SES (31%), 2=Medium-High-SES (33%), and 3=High-SES (11%).

Procedure

Research Collaborations

The data reported here are part of a joint research project between the TDSB and the University of Western Ontario (UWO), which was approved by the TDSB's External Research Review

¹ In many Canadian provinces, kindergarten is split into 'Junior' and 'Senior' Kindergarten. Junior Kindergarten is similar to what is sometimes referred to as 'preschool' elsewhere, as it is often relatively informal in overall structure and available to children who are 4 years old. Senior Kindergarten is more similar to what is referred to as kindergarten elsewhere. Senior Kindergarten tends to be more formally structured and involves the instruction of basic formal concepts in mathematics and other areas.

² Although participants were nested across 36 schools, intraclass correlations for each of the four regression models (described below) reveal that less than 5% of the variance in each outcome is attributable to variation across schools. As a multilevel approach thus contributed only minimally, and to simplify interpretation and facilitation of the results for a broader audience, we do not model school as a second level in our regression analyses.

³ School SES was estimated from median income, percentage of families below the Low-Income Measure, percentage of families on social assistance, percentage of parents without a high school diploma, percentage of parents with at least one university degree and percentage of lone-parent families.

Committee (ERRC). All data collection was conducted in collaboration with teachers, Early Childhood Educators (ECEs) and administrators in TDSB schools. The Board authorized TDSB's Research Department to collect assessment data and personal information for the purposes of the Board's educational planning. Parents of participating students were informed that classroom educators would be collecting the assessment data and that confidential student-level data would be kept within the TDSB's Research and Development Department. The TDSB's Research Department was authorized to share depersonalized data (stripped of any school or student identifiers) with related research partners for this study. Assessment materials were approved by the University of Western Ontario's Non-Medical Research Ethics Board.

Data Collection

Data were collected by the teachers and ECEs of the classrooms in which the testing took place. Teachers and ECEs were trained on administering the Numeracy Screener during an in-service work day. Administration of the Numeracy Screener was conducted during 15- 20 minute one-on-one testing sessions with the teacher/ECE and the student in a separate, quiet area at two time points: fall of 2014 and spring of 2015. The average interval between assessments was 191.99 days (range: 141-217 days, SD: 13.89).

In each testing session, the teacher/ECE went over a predefined set of instructions with the student. Task-specific instructions and general guidelines were printed in the booklet on the page before the start of each task. Before each task, the teacher/ECE went through several example items with the child to ensure that they understood the task and then explained to the child that, "You should try to complete as many problems as you can. You have two minutes. Work as fast as you can without making too many mistakes. If you make a mistake, draw an X through the mistake and put a new line through the right answer." A corrected answer counted as a correct trial. Finally, students were instructed to complete the items for each task in the order in which they were presented and not to skip any. After going over the instructions and practice items, the teacher/ECE started the timer, and the child began the task.

Numeracy Screener

The Numeracy Screener booklets were based on a design originally developed by Nosworthy et al. (2013). The booklets contain six basic numerical tasks, although only two (the numeral comparison and dot comparison tasks) are of relevance for the current study. Students completed

the numeral comparison task, followed by the dot comparison task. Both comparison tasks were made up of 72 items, with 12 items per page. Of the 72 items, 24 were comprised of two numerals/quantities within the subitizing range (1-4; hereafter referred to as small trials), 24 were comprised of two numerals/quantities outside of the subitizing range (6-9⁴; hereafter referred to as large trials), and 24 were comprised of one numeral/quantity within the subitizing range and the other outside of the subitizing range (trials of this type were not included in the analyses). Numerical parameters (not only size, but also ratio and distance) were counter-balanced across trials such that the child would have encountered roughly an equal number of ratios, sizes, and distances across both tasks, regardless of how far they got in each task. Continuous parameters such as dot area and overall contour length were also controlled for in the dot comparison task. On both tasks, participants completed as many items as possible within two-minutes. For more specific information on how continuous parameters were controlled, see Lyons et al. (2018).

Numerical Comparison

Children's symbolic number knowledge was assessed using the numeral comparison task ($\alpha = .83$; Lyons et al., 2018). Examples of small and large trials for this task are shown in Figure 1a. Before beginning the task, children were told, "In this task, your job is to decide which of the two numbers is bigger. Draw a line through the box with the number that means the most things." Children completed as many items as they could within two minutes.

Dot Comparison

Children's non-symbolic magnitude knowledge was assessed using the dot comparison task ($\alpha = .70$, Lyons et al., 2018). This task is thought to be a valid assessment of children's non-symbolic magnitude skills as it has been shown to reliably produce ratio effects (i.e., a decrease in performance as the ratio between the two quantities being compared approaches one; Lyons et al., 2018). An example of both a small and large trial for this task are shown in Figure 1b. Before beginning the task, children were told, "In this task, your job is to decide which of two boxes contains more dots. Draw a line through the box that has the most dots in it." Children were also instructed not to try and count the dots and were told "Instead, just look at the dots and try your

⁴ In order to equate the number of trials considered 'small' (1-4) and 'large' (6-9), we excluded trials that included the number 5 from analyses.

best to guess which side has more dots in it.” Children completed as many items as they could within two minutes.

Task Scoring

Raw scores were calculated as the total number of correct responses within the two-minute time limit. Scores were corrected for guessing (a child who randomly guessed on all 72 items would have received a score of 36), by using the standard adjustment: $A = C - [I/(P-1)]$, where A is the adjusted score, C is the number correct, I is the number incorrect, and P is the number of response options (Rowley & Traub, 1977). Using this adjustment, those who used a guessing strategy, on average, would receive a score of 0. For example, on a 4-item multiple-choice exam (where each choice is equally probable), those who randomly guessed on 20 items would, on average, receive a raw score (C in the equation above) of 5. Therefore, those who used a guessing strategy in this example would receive an adjusted score (A) of: $5 - [15/(4-1)] = 0$. In the current study, the task items only had two alternatives with equal probability of being correct (left and right quantity), so the equation for the adjusted score is essentially the number of items correct minus incorrect ($A = C - I$). Scores were calculated separately for subitizable and non-subitizable trials. Adjusted scores were used in all subsequent analyses.

Covariates

In all regression-based analyses we controlled for age at time 1 (years), sex, whether a child was born in Canada (0 or 1), school SES, percentage of days absent during the Senior Kindergarten school year ($M = 8.9\%$, $SD = 7.2\%$, $\text{min} = 0\%$, $\text{max} = 55\%$)⁵ and the testing interval (in days)⁶. All analyses were conducted using Stata 15 (StataCorp, 2017).

RESULTS

Task Descriptives

Table 1 reports the mean, standard deviation and minimum and maximum performance

⁵ Absentee rates were not available for three students. These students were assigned the average rate of 8.9%. Results did not differ if these children were excluded.

⁶ One of two testing dates was not available for 21 children and thus a testing interval could not be calculated. These children were assigned the average testing interval of 191.99 days. Results did not differ if these children were excluded.

(adjusted scores: correct – incorrect) on small and large trials for the numeral (NC) and dot comparison (DC) tasks in the fall and spring of the Senior Kindergarten year. Performance was above chance (> 0) for all conditions (all $ps < .001$). Also of note, children improved over the course of the Senior Kindergarten year across all conditions (all $ps < .001$). Table 2 displays the zero-order correlations between all conditions and both time-points.

Cross-Sectional Results

Here we analyzed the data in a cross-sectional manner to investigate whether symbolic and non-symbolic processing are more closely related within the subitizing range at the beginning and end of the Senior Kindergarten year. To do so, we calculated partial correlations between performance on the numeral and dot comparison tasks separately for small and large trials, controlling for age at time one, sex, whether a child was born in Canada, school SES, percentage of days absent during the senior kindergarten year and the testing interval. As these analyses were cross-sectional, they were conducted separately for fall and spring.

Results from the cross-sectional partial correlations are visualized in Figure 2. As can be seen in the figure, there was a significant, positive correlation between symbolic and non-symbolic processing for small and large trials in both the fall (Figure 2a) and spring (Figure 2b) of the kindergarten year. However, our main question concerned whether cross-format correlations significantly differed inside versus outside of the subitizing range. To test this, we compared small-trial with large-trial cross-format partial- r values using two standard Fisher's z -tests (one comparison at each time-point). Results showed that the cross-format relation between symbolic and non-symbolic processing was significantly stronger for small- relative to large-trials at both the beginning ($Z = 5.98, p < .001$) and end ($Z = 6.33, p < .001$) of the kindergarten year. These findings are consistent with the hypothesis that symbolic and non-symbolic processing are more closely related within the subitizing range.

Longitudinal Results

Here we analyzed data longitudinally to assess how symbolic and non-symbolic processing predict growth in one another, both within and outside the subitizing range. To test this, we asked whether symbolic processing at the beginning of Senior Kindergarten predicts growth in non-symbolic processing over the course of the school year, and vice versa. Crucially, we tested this for small- and large- trials separately. As such, we ran four separate regression models, key results from which are summarized in Figure 3 (full model results can be found in Table A-1 (small trials) and Table A-2 (large trials), Appendix A). All models included the covariates listed above as well as time 1 performance on the dependent variable. Controlling for time 1 performance removes time 1 variance from both the predictor and outcome, thus allowing one to assess the degree to which a given variable predicts *change* in the outcome. (Because children improved in all conditions from time 1 to time 2 on average – see Table 1 – we refer to this change here as ‘growth’.) By controlling for baseline performance, we are essentially controlling for the pre-existing relation between the outcome and the predictor (in addition to the autoregressive relation) which allows us to make directional inferences while also taking each individual’s intercept into account. This method of assessing growth has been used in prior studies on similar topics (e.g., Lyons et al., 2018; Mussolin et al., 2014). To assess whether symbolic skills predict growth in non-symbolic skills to a significantly different extent than non-symbolic skills predict growth in symbolic skills (both inside and outside of the subitizing range), we compared coefficients across models using seemingly unrelated estimation tests in Stata (SUEST).

Small-Trials

From Figure 3a, when looking specifically within the subitizing range, symbolic and non-symbolic skills in the fall are both predictive of growth in one another. Note that both relations are positive, indicating that stronger skills at time 1 in one skill predicts greater positive change in the other over the course of the school-year. While the partial r -value associated with the relation between NC and growth in DC appears larger than that associated with the relation between DC and growth in NC, results from the SUEST test indicated that these two relations are not significantly different from one another ($\chi^2_{539} = 0.46, p = .498$). Overall, these findings provide evidence for a bi-directional relation between symbolic and non-symbolic processing within the subitizing range.

Large-Trials

As can be seen in Figure 3b, when looking outside of the subitizing range, symbolic skills in the fall are a significant positive predictor of growth in non-symbolic abilities, but non-symbolic skills in the fall do not appear to significantly predict growth in symbolic abilities. Consistent with this, results from the SUEST test suggest that symbolic number skills are a significantly stronger predictor of growth in non-symbolic skills than the other way around ($\chi^2_{539} = 8.44, p = .004$). Taken together, these findings suggest that the unidirectional relation between symbolic and non-symbolic processing reported in prior studies (Lyons et al., 2018; Matejko & Ansari., 2016; Mussolin et al., 2014) may be specific to larger quantities outside of the subitizing range.

DISCUSSION

Recent longitudinal results have indicated that the relation between symbolic and non-symbolic processing may be unidirectional, with symbolic number skills predicting growth in non-symbolic processing but not the other way around (Lyons et al., 2018; Matejko & Ansari., 2016; Mussolin et al., 2014; Sasanguie et al., 2014). These findings suggest that while symbolic number abilities may play a role in the development of the AMS, the AMS plays perhaps only a limited role in the development of symbolic number skills (at least after the onset of formal education). However, despite evidence to suggest that the relation between symbolic and non-symbolic processing may be stronger within the subitizing range (Le Corre & Carey, 2007; Carey et al., 2017; Lyons & Beilock, 2018; Lyons et al., 2012), prior studies examining the longitudinal relations between symbolic and non-symbolic processing have failed to take numerical size into account. To address this gap in the literature, the aim of the current study was to investigate whether the relation between symbolic and non-symbolic processing differs inside versus outside of the subitizing range, both cross-sectionally and longitudinally, in a large sample of kindergarten children.

Cross-sectionally, we found that symbolic and non-symbolic processing were more strongly related within the subitizing range in both the fall and spring of the Senior Kindergarten year. Further, when looking at the longitudinal influence of these skills, we found evidence for a bi-directional relation between symbolic and non-symbolic processing specifically within the subitizing range, with non-symbolic skills in the fall predicting growth in symbolic processing to

roughly the same degree that symbolic skills in the fall predicted growth in non-symbolic processing. This is inconsistent with prior work that has observed an asymmetrical relation between these two systems in childhood, with symbolic skills predicting growth in non-symbolic skills but not the other way around (Lyons et al., 2018; Matejko & Ansari., 2016; Mussolin et al., 2014; Sasanguie et al., 2014). Where we *do* find evidence for the asymmetrical influence of symbolic number skills is on trials that included *large* quantities exclusively outside of the subitizing range, suggesting that these prior findings may have been driven by the inclusion of larger magnitudes. These findings inform the long-standing debate on the nature of the relation between symbolic and non-symbolic processing by suggesting that the influence of non-symbolic processing on symbolic number development may indeed persist beyond the onset of formal schooling, but that this influence appears to be limited to quantities within the subitizing range. Our results also refine recommendations for educators with respect to teaching basic numerical skills that are important for more complex math learning: school readiness in non-symbolic skills is likely to lead to growth in symbolic skills more so for small than large quantities; conversely, school readiness in symbolic skills is likely to lead to growth in non-symbolic skills regardless of numerical size.

Cross-Sectional Findings and Prior Literature

One hypothesis for why symbolic and non-symbolic processing may be more closely related within the subitizing range centers around the notion of exact representation: a key property of symbolic numbers (Núñez, 2017). Subitizable magnitudes are also thought to be processed exactly (Carey, 2009; Feigenson et al., 2002; Feigenson et al., 2004; Lipton & Spelke, 2004; Revkin et al., 2008; Uller et al., 1999), which may therefore explain the close relation between symbolic and non-symbolic number processing within the subitizing range. Evidence for a stronger link between symbolic and non-symbolic processing for small compared to large magnitudes has previously been observed in adults (Lyons et al., 2012; Lyons & Beilock, 2018).

Moreover, theories of symbolic number acquisition suggest that young children scaffold the acquisition of the meaning of number words via numbers within (but not outside) of the subitizing range (Le Corre & Carey, 2007; Carey et al., 2017). More specifically, through a series of experiments, Le Corre and Carey (2007) found that 3-4 year olds acquire the meaning of number words (the first number symbols most children learn) by directly linking them with

sets of objects but only within the subitizing range whose exact numerical value can be apprehended via parallel individuation. The meanings of larger number words referring to quantities outside the subitizing range are later inferred only by extension of counting principles, without recourse to their corresponding non-symbolic (AMS) counterparts (Carey et al., 2017). Others however, have challenged the view put forward by Le Corre and Carey and provide evidence to suggest that the AMS may in fact play a role in the acquisition of number words (Wagner and Johnson, 2011; VanMarle et al., 2018). While the current findings cannot directly address this debate as they do not inform how children initially acquire number words, they are broadly aligned – albeit later in development -- with the view put forward by Le Corre and Carey who suggest that symbolic and non-symbolic magnitude systems may be primarily linked within the subitizing range.

Overall, our cross-sectional findings contribute to the literature in two ways. For one, we provide evidence for a stronger link between symbolic and non-symbolic processing within the subitizing range in children at the outset of formal education, *after* they have acquired the meaning of number symbols. Further, we provide evidence that symbolic and non-symbolic processing are more closely related at both the beginning and end of the kindergarten year, which is to say that our findings are developmentally stable across the first year of formal schooling.

Longitudinal Findings and Prior Literature

While there is a growing body of literature indicating that symbolic and non-symbolic processing may be more closely related within the subitizing range, the current study is the first to suggest that the stronger relation between subitizable quantities and numbers symbols may have implications for growth in these skills. Prior studies investigating the longitudinal relations between symbolic and non-symbolic processing in early childhood report evidence of a unidirectional relation between these two systems in which symbolic number skills predict growth in non-symbolic processing but not the other way around (Lyons et al., 2018, Matejko & Ansari, 2016; Mussolin et al., 2014). Findings from these studies converge around the conclusion that non-symbolic processing plays perhaps only a limited (if any) role in the development of symbolic number skills. However, findings from the current study suggest that such a conclusion may be premature. More specifically, we find evidence for a *bidirectional* relation between symbolic and non-symbolic processing when we look specifically within the subitizing range.

The fact that we observe a positive relation between non-symbolic processing and growth in symbolic number skills within the subitizing range suggests that non-symbolic processing does in fact play some role in the development of symbolic number skills even after children have acquired a basic understanding of the meaning of number symbols. Where we find evidence for the previously reported unidirectional relation between symbolic and non-symbolic number skills is for larger quantities, suggesting that the prior findings may have been partially driven by the inclusion of trials involving larger numbers.

In light of this new evidence, prior conclusions concerning the relation between symbolic and non-symbolic processing should be updated to take numerical size into account. In the prior literature, two explanations have been put forward to explain why symbolic number skills appear to predict growth in non-symbolic processing but not the other way around. For one, Piazza et al. (2013) and Mussolin et al. (2014) have both suggested that practice with numbers symbols may help to improve the representational precision of the AMS. In other words, gaining more experience with exact representations of number (i.e. number symbols) may help to refine non-symbolic representations of magnitude, thereby rendering them easier to discriminate between. In contrast, Lyons et al. (2018) suggested that symbolic number skills may predict growth in non-symbolic processing simply as a result of near-transfer of learning how to perform comparison tasks in general. Specifically, the authors note that over the course of the school year, children demonstrate the greatest amount of improvement on the number comparison task and suggest that learning how to do this one type of comparison task transfers to performance on other types of comparison tasks. The results of the current paper suggest that both accounts may have the right of it, but in a way that depends on the size of the quantities in question.

On the one hand, we suggest that the bidirectional relation we observed here for *subitizable* quantities is more consistent with the Lyons et al. (2018) proposal based on task-level near-transfer, at least within this range. As was emphasized in the Introduction, subitizable magnitudes and symbolic numbers are thought to share the property of exact representation (Carey, 2009; Feigenson et al., 2002; Feigenson et al., 2004; Lipton & Spelke, 2004; Revkin et al., 2008; Uller et al., 1999). As both forms of representation in this range are exact, it is hard to see how there is significant room for improvement in terms of representational precision, thus making it difficult to reconcile these results with the interpretation put forward by Piazza et al.

(2013) and Mussolin (2014). Instead, improvement is perhaps more likely to be due to how these quantities are processed rather than how they are represented. Hence, the fact that subitizable quantities and symbolic numbers are already aligned in terms of (very high) representational precision may in fact facilitate task-level transfer in both directions for subitizable trials. In other words, practice with one exact quantity discrimination task is likely to improve performance on another exact quantity discrimination task and vice versa.

Comparatively, one could imagine a reduction in the ease of task-level transfer for larger quantity trials where there is thought to be a greater degree of representational misalignment between symbolic and non-symbolic processing. In other words, the degree of task-level transfer between an exact quantity discrimination task (symbolic number comparison) and an approximate quantity discrimination task (large quantity comparison) may be lower than the degree of task-level transfer between two exact quantity discrimination tasks. As such, the task-level transfer view put forward by Lyons et al. (2018) may be less compelling when it comes to larger quantities. And indeed, our results here indicate that the unidirectional result in Lyons et al. (2018; as well as in Matejko & Ansari, 2016 and Mussolin et al., 2014) may have been driven primarily by the larger quantities. That is, our results for *large* quantities appear to be more consistent with the view put forth by Piazza et al. (2013) and Mussolin (2014). More specifically, while symbolic numbers are represented exactly, large magnitudes are thought to be represented approximately. As such, contrary to subitizable quantities, there is room for growth in the representational acuity of non-symbolic quantities outside of the subitizing range. Therefore, practice with exact representation of number may help to refine non-symbolic representations of larger magnitudes, thereby rendering them easier to discriminate between.

A third and previously undiscussed proposal for the unidirectional influence of symbolic to non-symbolic large quantities is that experience with large symbolic numbers cues young children to the very notion that large quantities *can* be represented exactly. Prior to learning number symbols, children experience small quantities in exact form, but not large quantities. Recent work suggests that symbolic quantities are first understood in the subitizing range, and, crucially, this understanding is generalized to larger symbolic numbers without direct reference to their approximate analogues (Le Corre & Carey, 2007; Carey et al., 2017). Hence, we suggest that once children come to the realization that large non-symbolic quantities *can* be represented

exactly may change the way they approach the dot comparison task especially with respect to large-quantity trials. This in turn leads to an especially strong unidirectional effect from symbolic to non-symbolic growth for trials outside the subitizing range. Therefore, it may not be that experience with symbolic numbers is refining the representational acuity of the AMS, but that experience with exact representation of large numbers changes the way children understand the very boundaries of what large quantities can be. Finally, we should note that this view is not mutually exclusive with those discussed above. While future work is surely needed to unpack why we observe a unidirectional relation from symbolic to non-symbolic growth, the results presented here clearly indicate that such work would do well to take numerical size into account – in particular, whether quantities are within or outside the subitizing range.

Practical Implications

As evidenced above, the findings from the current study primarily inform theory. However, given the centrality of symbolic number skills in early education, our findings may have some educational implications. Primarily, our results suggest that when teaching about non-symbolic magnitudes in the classroom, a focus on subitizable, rather than larger, magnitudes may carry more weight in terms of symbolic number development. Further, our results suggest that a focus on symbolic numbers in school may be beneficial for the development of *both* symbolic and non-symbolic magnitude skills, regardless of numerical size. However, it is important to note that the mechanism mediating the unidirectional relation between symbolic and non-symbolic processing for larger magnitudes, in which symbolic number abilities predict growth in non-symbolic number skills, remains largely unknown.

Conclusion

The current work provides the first evidence of a particularly strong link between symbolic and non-symbolic number processing specifically within the subitizing range in children at the outset of schooling. Prior work has shown this link during the acquisition of number words (Le Corre & Carey, 2007; Carey et al., 2017); here we show both cross-sectionally and longitudinally that this link persists even after children have acquired the meaning of number symbols. Moreover, evidence for a bidirectional relation link over the course of kindergarten indicates that prior researchers may have been too quick to dismiss the potential role of – at least subitizable – non-symbolic quantities in scaffolding growth in symbolic number skills. Instead, evidence for a

unidirectional link (symbolic→non-symbolic, but not non-symbolic→symbolic) appears to be specific to larger quantities outside the subitizing range – a result broadly consistent with both work in how toddlers acquire number words (Le Corre & Carey, 2007; Carey et al., 2017) and neuroimaging work with adults (Lyons & Beilock, 2018). Finally, these results may have implications for how kindergarten teachers choose the quantities to focus on when emphasizing the link between symbolic and non-symbolic quantities. Together, the results presented here help unify several disparate lines of research across multiple methods and age-groups to converge on the notion that symbolic and non-symbolic quantities are closely linked within, but not outside of the subitizing range.

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REFERENCES

- Ansari, D., Lyons, I. M., van Eimeren, L., & Xu, F. (2007). Linking visual attention and number processing in the brain: The role of the temporo-parietal junction in small and large symbolic and non-symbolic number comparison. *Journal of Cognitive Neuroscience*, *19*(11), 1845-1853. doi: 10.1162/jocn.2007.19.11.1845
- Bailey, D. H., Duncan, G. J., Watts, T., Clements, D. H., & Sarama, J. (2018). Risky business: Correlation and causation in longitudinal studies of skill development. *American Psychologist*, *73*(1), 81. doi: 10.1037/amp0000146
- Bulthé, J., De Smedt, B., & de Beeck, H. O. (2014). Format-dependent representations of symbolic and non-symbolic numbers in the human cortex as revealed by multi-voxel pattern analyses. *NeuroImage*, *87*, 311-322. doi: 10.1016/j.neuroimage.2013.10.049
- Burr, D. C., Turi, M., & Anobile, G. (2010). Subitizing but not estimation of numerosity requires attentional resources. *Journal of Vision*, *10*(6), 20-20. doi: 10.1167/10.6.20
- Cantlon, J. F. (2012). Math, monkeys, and the developing brain. *Proceedings of the National Academy of Sciences*, *109*(Supplement 1), 10725-10732. doi: 10.1073/pnas.1201893109
- Carey, S. (2009). Where our number concepts come from. *The Journal of Philosophy*, *106*(4), 220. doi: 10.5840/jphil2009106418
- Carey, S., Shusterman, A., Howard, P., & Distefano, R. (2017). Do analog number representations underlie the meanings of young children's verbal numerals? *Cognition*, *168*, 243-255. doi: 10.1016/j.cognition.2017.06.022
- Chu, F. W., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology*, *132*, 205-212. doi: 10.1016/j.jecp.2015.01.006
- Chu, F. W., vanMarle, K., & Geary, D. C. (2016). Predicting children's reading and mathematics achievement from early quantitative knowledge and domain-general cognitive abilities. *Frontiers in psychology*, *7*, 775. doi: fpsyg.2016.00775
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42. doi: 10.1016/0010-0277(92)90049-n
- Dehaene, S. (2007). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. *Sensorimotor Foundations of Higher Cognition*, *22*, 527-574. doi: 10.1093/acprof:oso/9780199231447.003.0024

- De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, 2(2), 48-55. doi: 10.1016/j.tine.2013.06.001
- De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, 103(4), 469-479. doi: 10.1016/j.jecp.2009.01.010
- Egeth, H. E., Leonard, C. J., & Palomares, M. (2008). The role of attention in subitizing: Is the magical number 1? *Visual Cognition*, 16(4), 463-473. doi: 10.1080/13506280801937939
- Elliott, L., Feigenson, L., Halberda, J., & Libertus, M. E. (2019). Bidirectional, Longitudinal Associations Between Math Ability and Approximate Number System Precision in Childhood. *Journal of Cognition and Development*, 20(1), 56-74. doi: 10.1080/15248372.2018.1551218
- Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object files versus analog magnitudes. *Psychological Science*, 13(2), 150-156. doi: 10.1111/1467-9280.00427
- Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314. doi: 10.1016/j.tics.2004.05.002
- Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives*, 7(2), 74-79. doi: 10.1111/cdep.12019
- Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptual understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological science*, 29(2), 191-205. doi: 10.1177/0956797617729817
- Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkman, J. (1949). The discrimination of visual number. *The American Journal of Psychology*, 62(4), 498-525. doi: 10.2307/1418556
- Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, 105(2), 395-438. doi: 10.1016/j.cognition.2006.10.005
- Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Numerical approximation abilities correlate with and predict informal but not formal mathematics abilities. *Journal of Experimental Child Psychology*, 116(4), 829-838. doi: 10.1016/j.jecp.2013.08.003

- Lipton, J. S., & Spelke, E. S. (2004). Discrimination of large and small numerosities by human infants. *Infancy*, 5(3), 271-290. doi: 10.1207/s15327078in0503_2
- Lyons, I. M., Ansari, D., & Beilock, S. L. (2012). Symbolic estrangement: Evidence against a strong association between numerical symbols and the quantities they represent. *Journal of Experimental Psychology: General*, 141(4), 635-641. doi: 10.1037/a0027248
- Lyons, I. M., Ansari, D., & Beilock, S. L. (2015). Qualitatively different coding of symbolic and non-symbolic numbers in the human brain. *Human Brain Mapping*, 36(2), 475-488. doi: 10.1002/hbm.22641
- Lyons, I. M., Bugden, S., Zheng, S., De Jesus, S., & Ansari, D. (2018). Symbolic number skills predict growth in non-symbolic number skills in kindergarteners. *Developmental psychology*, 54(3), 440. doi: 10.31219/osf.io/m7sgz
- Lyons, I. M., & Beilock, S. L. (2018). Characterizing the neural coding of symbolic quantities. *NeuroImage*. doi: 10.1016/j.neuroimage.2018.05.062
- Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, 17(5), 714-726. doi: 10.1111/desc.12152
- Mandler, G., & Shebo, B. J. (1982). Subitizing: an analysis of its component processes. *Journal of Experimental Psychology: General*, 111(1), 1. doi: 10.1037/0096-3445.111.1.1
- Matejko, A. A., & Ansari, D. (2016). Trajectories of symbolic and non-symbolic magnitude processing in the first year of formal schooling. *PloS one*, 11(3), e0149863. doi: 10.1371/journal.pone.0149863
- Mussolin, C., Nys, J., Content, A., & Leybaert, J. (2014). Symbolic number abilities predict later approximate number system acuity in preschool children. *PLoS One*, 9(3), e91839. doi: 10.1016/j.tine.2012.09.003
- Nosworthy, N., Bugden, S., Archibald, L., Evans, B., & Ansari, D. (2013). A two-minute paper-and-pencil test of symbolic and non-symbolic numerical magnitude processing explains variability in primary school children's arithmetic competence. *PloS one*, 8(7), e67918. doi: 10.1371/journal.pone.0067918
- Núñez, R. E. (2017). Is there really an evolved capacity for number? *Trends in cognitive sciences*, 21(6), 409-424. doi: 10.1016/j.tics.2017.03.005

- Piazza, M. (2011). Neurocognitive start-up tools for symbolic number representations. In *Space, Time and Number in the Brain* (pp. 267-285). doi: 10.1016/j.tics.2017.03.005
- Piazza, M., Fumarola, A., Chinello, A., & Melcher, D. (2011). Subitizing reflects visuo-spatial object individuation capacity. *Cognition*, 121(1), 147-153. doi: 10.1016/j.cognition.2011.05.007
- Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal approximate number system. *Psychological science*, 24(6), 1037-1043. doi: 10.1177/09567
- Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does subitizing reflect numerical estimation? *Psychological science*, 19(6), 607-614. doi: 10.1111/j.1467-9280.2008.02130.x
- Rowley, G. L., & Traub, R. E. (1977). Formula scoring, number-right scoring, and test-taking strategy. *Journal of Educational Measurement*, 14(1), 15-22. doi: 10.1111/j.1745-3984.1977.tb00024.x
- Sasanguie, D., Defever, E., Maertens, B., & Reynvoet, B. (2014). The approximate number system is not predictive for symbolic number processing in kindergarteners. *The Quarterly Journal of Experimental Psychology*, 67(2), 271-280. doi: 10.1080/17470218.2013.803581
- Shusterman, A., Slusser, E., Halberda, J., & Odic, D. (2016). Acquisition of the cardinal principle coincides with improvement in approximate number system acuity in preschoolers. *PloS one*, 11(4), e0153072. doi: 10.1371/journal.pone.0153072
- Suárez-Pellicioni, M., & Booth, J. R. (2018). Fluency in symbolic arithmetic refines the approximate number system in parietal cortex. *Human brain mapping*, 39(10), 3956-3971. doi: 10.1002/hbm.24223
- Toll, S. W., Van Viersen, S., Kroesbergen, E. H., & Van Luit, J. E. (2015). The development of (non-) symbolic comparison skills throughout kindergarten and their relations with basic mathematical skills. *Learning and Individual Differences*, 38, 10-17. doi: 10.1016/j.lindif.2014.12.006
- Uller, C., Carey, S., Huntley-Fenner, G., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive development*, 14(1), 1-36. doi:10.1016/s0885-2014(99)80016-1
- VanMarle, K., Chu, F. W., Mou, Y., Seok, J. H., Rouder, J., & Geary, D. C. (2018). Attaching meaning to the number words: Contributions of the object tracking and approximate number systems. *Developmental Science*, 21(1), e12495. doi: 10.1111/desc.12495
- Wagner, J. B., & Johnson, S. C. (2011). An association between understanding cardinality and analog magnitude representations in preschoolers. *Cognition*, 119, 10-22. doi:10.1016/j.cognition.2010.11.014

Watson, D. G., Maylor, E. A., & Bruce, L. A. (2007). The role of eye movements in subitizing and counting. *Journal of Experimental Psychology: Human Perception and Performance*, 33(6), 1389-1399. doi: 10.1037/0096-1523.33.6.1389

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74(1), B1-B11. doi: 10.1016/s0010-0277(99)00066-9

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science*, 8(1), 88-101. doi: 10.1111/j.1467-7687.2005.00395.x

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TABLES**Table 1 – Task Descriptives**

	Fall		Spring	
	Mean	Std. Dev	Mean	Std. Dev
NC				
<i>Small</i>	6.98	5.30	10.93	4.96
<i>Large</i>	5.46	5.82	9.73	5.99
DC				
<i>Small</i>	7.61	4.21	10.44	4.31
<i>Large</i>	2.30	2.92	3.94	3.74

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Table 2 – Zero-order Correlations

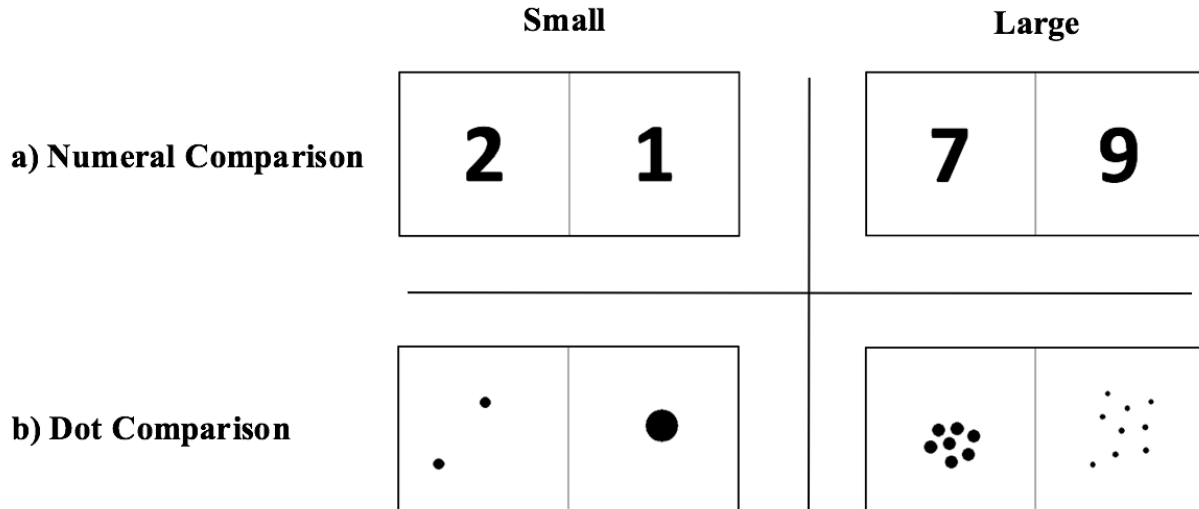
		Fall				Spring			
		NCS	DCS	NCL	DCL	NCS	DCS	NCL	DCL
Fall	NCS	-	0.62	0.83	0.28	0.64	0.50	0.66	0.46
	DCS	0.62	-	0.51	0.27	0.51	0.49	0.48	0.30
	NCL	0.83	0.51	-	0.32	0.61	0.45	0.68	0.47
	DCL	0.28	0.27	0.32	-	0.19	0.14	0.24	0.32
Spring	NCS	0.64	0.51	0.61	0.19	-	0.70	0.85	0.46
	DCS	0.50	0.49	0.45	0.14	0.70	-	0.60	0.40
	NCL	0.66	0.48	0.68	0.24	0.85	0.60	-	0.46
	DCL	0.46	0.30	0.47	0.32	0.46	0.40	0.46	-

Table 2 shows zero-order correlations between all conditions across both time-points. Abbreviations: DCS = dot comparison (small trials), DCL = dot comparison (large trials), NCS = number comparison (small trials), NCL = number comparison (large trials). All correlations were significant at the $p < .001$ level

FIGURES

Figure 1 – Task Examples

Figure 1 shows examples of small and large trials for the numeral and dot comparison tasks.



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Figure 2 – Cross-Sectional Results

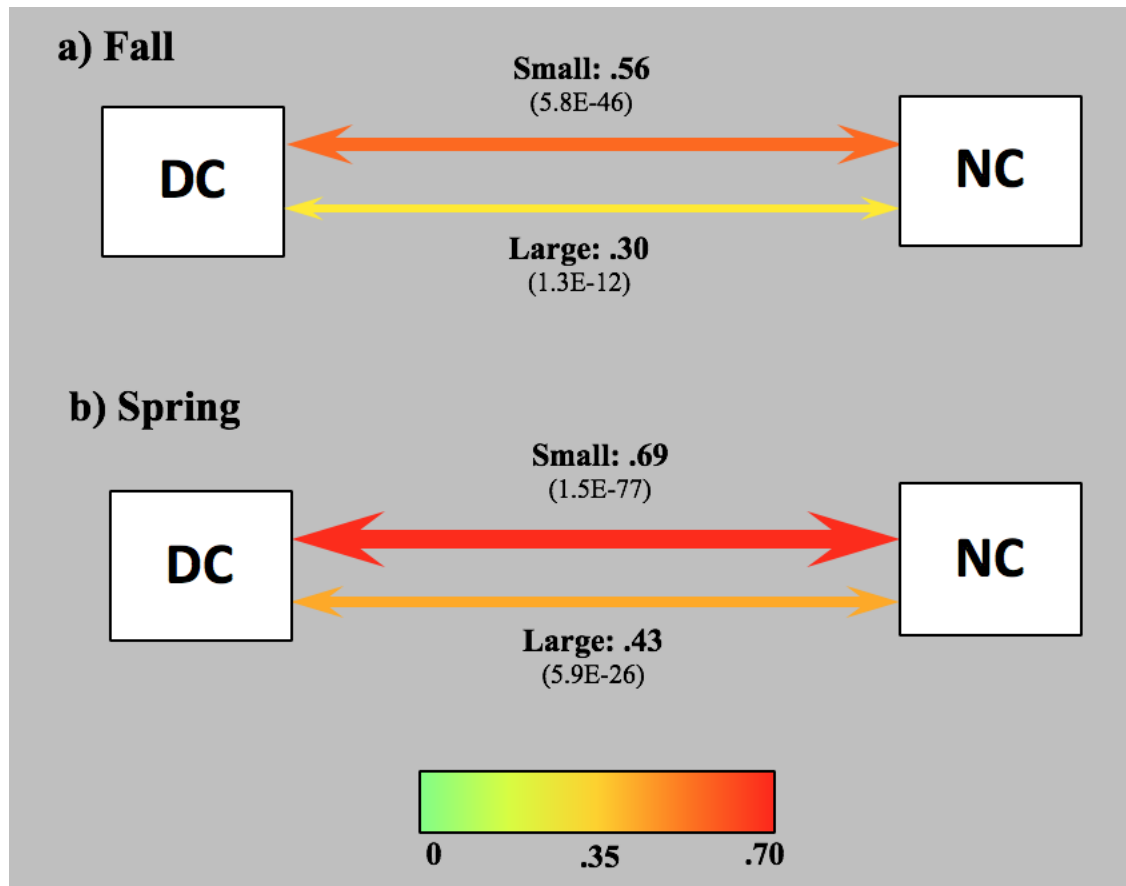


Figure 2 visualizes the results from the cross-sectional partial correlations between performance on the small and large trials of the numeral and dot comparison tasks in both the fall and spring of the kindergarten year. In Figure 2a and 2b, the correlation between the small trials of the numeral and dot comparison is indicated by the bidirectional arrow on the top, while the correlation between large trials of the number and dot comparison task is indicated by the bidirectional arrow on the bottom. The size and color of the arrows reflect the magnitude of the partial r -values. Exact values are in bold; numbers in parentheses are corresponding p -values. Abbreviations: DC: dot comparison, NC: numeral comparison.

Figure 3 – Longitudinal Results

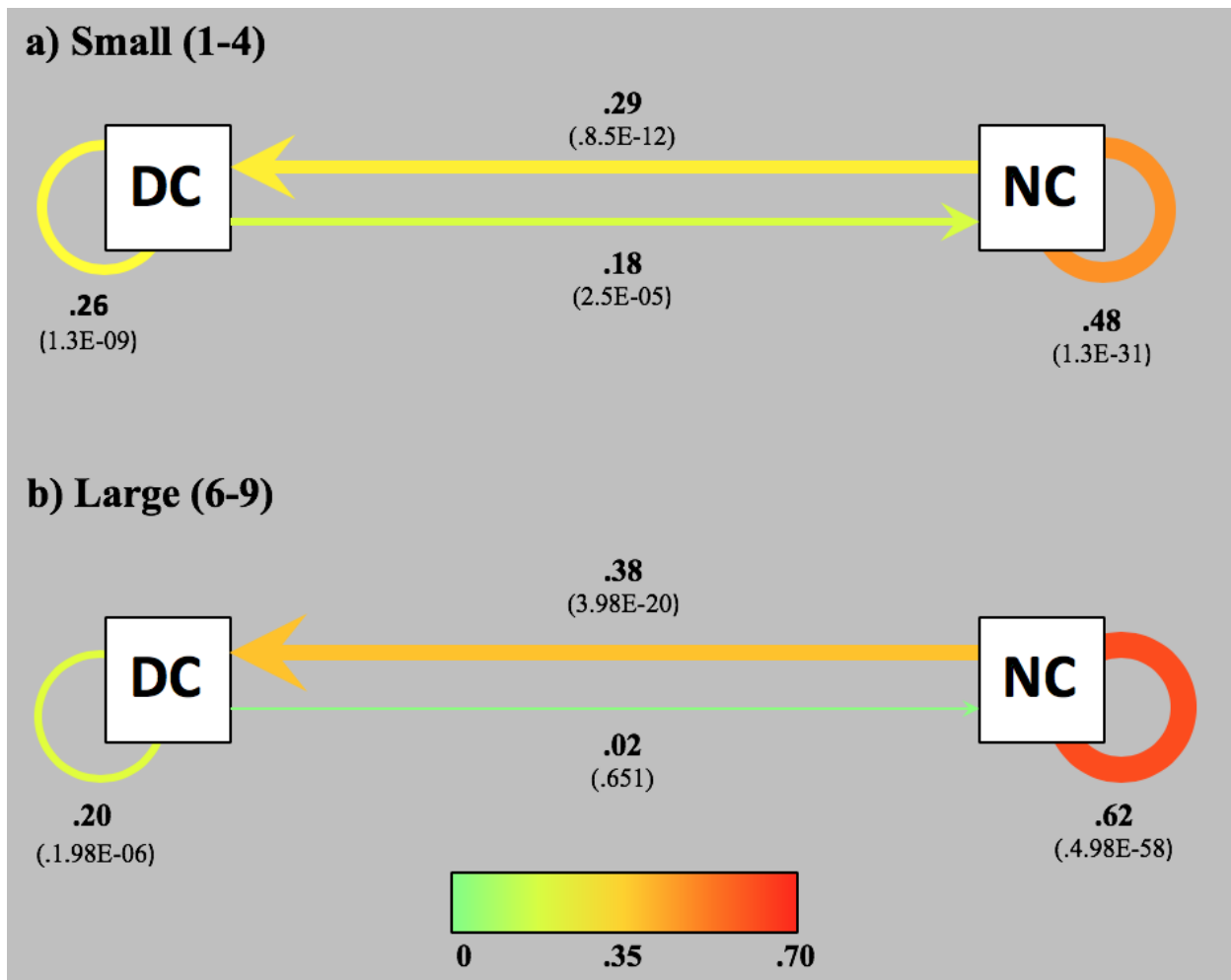


Figure 3 visualizes key results of primary theoretical interest from the longitudinal regression models (see Appendix A for full regression details), predicting growth in symbolic and non-symbolic processing for small and large trials. Arrows going from one task to the other indicates the unique relation between the originating task in the fall and growth in the outcome (time 2 performance controlling for time 1 performance). Circular arrows indicate autoregressive effects (the unique relation between a given task across time-points). The size and color of the arrows reflect the magnitude of partial r -values. Exact values are in bold; numbers in parentheses are corresponding p -values. Abbreviations: DC: dot comparison, NC: numeral comparison.

APPENDIX 1

Table A-1 – Longitudinal Regression Results for Small-Trials

<u>IV (Predictor)</u>	<u>Outcome: DC (Spring)</u>			<u>Outcome: NC (Spring)</u>		
	<i>b</i>	<i>se</i>	<i>p</i>	<i>b</i>	<i>se</i>	<i>p</i>
DC (Fall)	0.29	0.05	1.3E-09	0.18	0.05	2.5E-05
NC (Fall)	0.33	0.04	8.5E-12	0.53	0.04	1.3E-31
Age	-0.06	0.55	.101	0.00	0.58	.901
Gender	-0.05	0.31	.204	0.05	0.33	.116
Born in Canada	0.06	0.47	.118	0.03	0.49	.428
Percent Absent	0.00	2.21	.941	0.01	2.30	.843
School SES	0.04	0.17	.268	0.01	0.18	.774
Testing Interval	-0.04	0.01	.225	0.02	0.01	.540
Constant	12.66	3.51	3.4E-04	3.21	3.66	.381
Adjusted R^2	0.30			0.43		
<i>df-residual</i>	531			531		

Table A-1 shows standardized regression coefficients for models predicting growth in non-symbolic and symbolic performance for small-trials. Predictors of no interest (those not shown in Figure 3a) are greyed out. Abbreviations: DC: dot comparison, NC: numeral comparison.

Table A-2 – Longitudinal Regression Results for Large-Trials

<u>IV (Predictor)</u>	<u>Outcome: DC (Spring)</u>			<u>Outcome: NC (Spring)</u>		
	<i>b</i>	<i>se</i>	<i>p</i>	<i>b</i>	<i>se</i>	<i>p</i>
DC (Fall)	0.19	0.05	1.98E-06	0.02	0.07	.651
NC (Fall)	0.40	0.03	3.98E-20	0.65	0.04	4.98E-58
Age	0.08	0.49	.043	0.05	0.67	.125
Gender	-0.01	0.28	.836	0.02	0.39	.487
Born in Canada	-0.02	0.42	.586	0.02	0.58	.583
Percent Absent	0.00	1.99	.955	-0.01	2.71	.813
School SES	-0.04	0.15	.323	0.04	0.21	.278
Testing Interval	-0.02	0.01	.612	0.03	0.01	.351
Constant	-1.60	3.17	.615	-2.80	4.33	.517
Adjusted R^2	0.25			0.46		
<i>df-residual</i>	531			531		

Table A-2 shows standardized regression coefficients for models predicting growth in non-symbolic and symbolic performance for large-trials. Predictors of no interest (those not shown in Figure 3b) are greyed out. Abbreviations: DC: dot comparison, NC: numeral comparison.